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MHD Oscillatory Flow of Jeffrey Fluid in an Indented Artery with Heat Source

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Authors' contributions

This work was carried out in collaboration between all authors. Authors CIC and KWB designed the study, performed the mathematical formulations. Authors EA and KWB proffered the solutions, presented the plots and interpreted the results. Author KWB managed literature searches and wrote the first draft of the manuscript. All authors read and approved the final manuscript.

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Abstract

In this article we studied blood flow through an indented artery and assumed blood to be Jeffrey fluid. The study investigates the influence of heat transfer on the flow profile of Jeffrey fluid in an indented artery. Also, the formulated governing equations are transformed into coupled Bessel differential equation and solved analytically. The effects of various physical parameters such as Prandtl number of blood, $Pr = 21$, Grashof number, Gr, Darcy number, Da, Hartmann number M, Reynold number, Re, as well as the Jeffrey parameter, ξ_1 and a constant parameter, ξ on the velocity profile and temperature profile. The results are discussed in detail with the graphs obtained using Mathematica version 10.3.

Keywords: Oscillation; MHD; heat transfer; Darcy number; Jeffrey fluid; artery.

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NOMENCLATURES

(u', v', w') : *Dimensional velocity components*
z' : *Dimensional axisymetric direction of the flow*
r' : *Dimensional radius of the artery*
t' : *Dimensional time*
T_w : *Dimensional wall temperature*
T' : *Dimensional Fluid Temperature*
T_∞ : *Fluid temperature at the stent region*
M : *Hartmann number*
K: *Porosity*
Pr : *Prandtl number*
Gr : *Grashoff number*
C_p : *Specific heat capacity of the fluid at constant pressure*
q'_r : *Radiative heat flux*
Da : *Darcy number*
$R(z)$: *Radius of the indented region of the artery*
R_0 : *Radius of the normal artery*
L_0 : *Length of the artery under investigation*
a : $a = \frac{R}{R_0}$
ξ_1 : *Ratio of relaxation to retardation time*
ξ_2 : *Retardation time*
d : *Distance of the onset of stenosis*
k_T : *Thermal conductivity*
$w_0(r)$: *Dimensionless velocity*
$w(r,t)$: *Velocity profile of the fluid*

GREEK SYMBOLS

ν : *Kinematic viscosity*
μ : *Dynamic viscosity of the fluid*
g : *Acceleration due to gravity*
ρ : *Density of the fluid*
σ_c : *Electrical conductivity*
ω : *Frequency parameter*
β_T : *Coefficient of volume expansion due to temperature*
β, λ : *Modified radiation parameters*
P_0 : *Oscillatory pressure*
θ_0 : *Dimensionless temperature*
$\theta(r,t)$: *Temperature profile*

1 Introduction

Numerous cardiovascular sicknesses, especially atherosclerosis (therapeutically called stenosis), observed to be responsible for deaths in developed and developing countries, are closely related to the nature of blood movement and the dynamic behaviour of blood vessels, are firmly identified with the idea of blood development and the dynamic conduct of vessels. Stenosis implies the anomalous and unnatural development in the lumen of an artery that creates at different areas in the cardiovascular framework under horrible conditions.

From medical surveys, it is well known that more than 80% of the total deaths are due to the diseases of blood vessel walls [1]. Among them, stenosis is a dangerous disease that is caused due to deposition of cholesterol and some other substances on the endothelium and by the proliferation of connective tissues in the arterial wall. The reason for the formation of stenosis in the lumen of an artery is not known but its effect over the flow characteristics has been studied by many researchers. But deposition of various substances such as cholesterol and other fatty materials called plaque on the endothelium of the arterial wall and proliferation of connective tissues are believed to be the factors that accelerate the formation of stenosis [2]. Cardiovascular diseases such as stroke, heart attack, heart failure are associated with some form of an abnormal flow of blood in stenotic arteries [3]. Heat transfer effect on laminar flow between parallel plates under the action of transverse magnetic field was studied by Nigam and Singh [4]. Soundalgekar and Bhat [5] have investigated the approximate analysis of an oscillatory MHD channel flow and heat transfer under transverse magnetic field. The transient and steady velocity, the transient and steady magnetic field was shown graphically. MHD flow of viscous fluid between two parallel plates with heat transfer was discussed by Attia, and Kotb [6]. Raptis et al. [7] have analyzed the hydromagnetic free convection flow through a porous medium between two parallel plates. Aldoss et al. [8] have studied mixed convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field. Makinde and Mhone [9] have considered heat transfer to MHD oscillatory flow in a channel filled with porous medium. Mostafa [10] have studied thermal radiation effect on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity. Unsteady heat transfer to MHD oscillatory flow through a porous medium under slip condition was investigated by Hamza et al. [11].

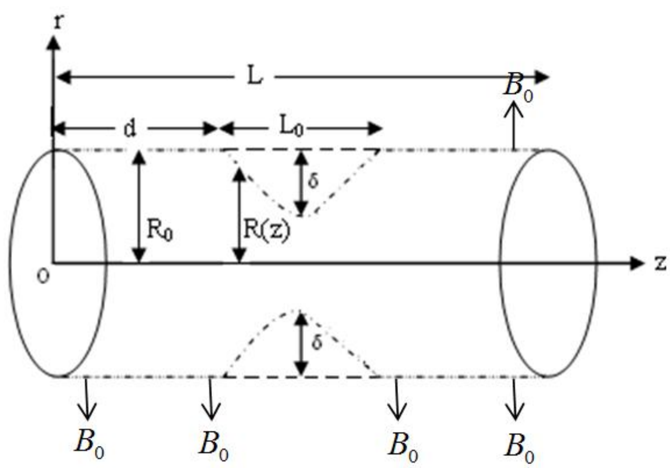
Moreover, the non-Newtonian fluids are more appropriate than Newtonian fluids in many practical applications.

Examples of such fluids include certain oils, lubricants, mud, shampoo, ketchup, blood, cosmetic products, polymers and many others. Unlike the viscous fluids, all the non-Newtonian fluids (in terms of their diverse characteristics) cannot be described by a single constitutive relationship. Hence, several models of non-Newtonian fluids are proposed in the literature. Al Khatib and Wilson [12] have studied the Poiseuille flow of a yield stress fluid in a channel. Flow of a visco-plastic fluid in a channel of slowly varying width was studied by Frigaard and Ryan [13]. Ali and Asghar [14] have analyzed by oscillatory channel flow for non-Newtonian fluid. Choudhury and Das [15] have studied the effect of heat transfer on MHD oscillatory viscoelastic fluid flow in a channel through a porous medium. [16] investigated an axisymmetric blood flow through an axially non-symmetreic but radially symmetric mild stenosis tapered artery. To estimate the effect of the stenosis shape, a suitable geometry has been considered such that the axial shape of the stenosis can be changed easily just by varying a parameter. The model is also used to study the effect of the taper angle ϕ. Akbar et al. [17] studied a non-Newtonian fluid model for blood flow through a tapered artery with a stenosis by assuming blood as Jeffrey fluid. The main purpose of our study was to study Jeffrey fluid model for blood flow through a tapered artery with a stenosis, Jeffrey fluid model is a non-Newtonian fluid model in which we consider convective derivative instead of time derivative. It is capable of describing the phenomena of relaxation and retardation time. The Jeffrey fluid has two parameters, the relaxation time λ_1 and retardation time λ_2. Perturbation method is used to solve the resulting equations. The effects of non-Newtonian nature of blood on velocity profile, wall shear stress, shearing stress at the stenosis throat, and impedance of the artery are discussed.

In view of these we studied the effect of heat transfer on MHD oscillatory flow of a Jeffrey fluid (blood) in an indented artery. The expressions are obtained for velocity and temperature analytically. The effects of various emerging parameters on the velocity and temperature are discussed graphically in detail.

2 Mathematical Formulation

We consider MHD oscillatory flow of Jeffrey fluid in an indented artery of radius R_0. A uniform magnetic field B_0 being applied in the transverse direction to the flow. The wall of the artery is maintained at a temperature T_w. We also consider the cylindrical coordinates (r, θ, z) in such as way that $r \neq 0$ is the axis of symmetry. The flow is considered as axiallialy symmetric and fully developed. The schematic of the flow is shown in figure below.



The equation of S for Jeffrey fluid is

$$S = \frac{\mu}{1+\xi_1}(\dot{\gamma} + \xi_2 \ddot{\gamma}) \tag{2.1}$$

where μ is the dynamic viscosity, ξ_1 is the ratio of relaxation to retardation times, ξ_2 is the retardation time, $\dot{\gamma}$ is differentiation of the shear rate with time.

The governing equations of the flow are given by

$$\rho \frac{\partial w'}{\partial t'} = -\frac{\partial p'}{\partial z'} + \frac{1}{r'}\frac{\partial}{\partial r'}\left(rS\right) - \sigma_c B_0'^2 w' + \rho \beta_T (T' - T_\infty)\vec{g}\vec{k} - \frac{\mu}{\mathrm{K}} w' \tag{2.2}$$

$$\frac{\partial T'}{\partial t'} = \frac{k_T}{\rho C_p}\left(\frac{\partial T'^2}{\partial r'^2} + \frac{1}{r'}\frac{\partial T'}{\partial r'}\right) \tag{2.3}$$

where ρ is the fluid density, μ is the fluid viscosity, p is the pressure, w' is the velocity component in z direction, K is the permeability of the porous medium, g is the acceleration due to gravity, σ_c is the electrical conductivity of the fluid, β_T is the coefficient of thermal expansion, T' is the temperature, k_T is the thermal conductivity and C_p is the specific heat at constant pressure.

The appropriate boundary conditions are

$$\begin{aligned} & w'=0\ ,\ \ T'=T_w \text{ at } r'=R_0(\mathrm{z}) \\ & w'=0,\ \ T'=T_\infty \text{ at } r'=R(\mathrm{z}) \end{aligned} \tag{2.4}$$

We shall now introduce the following non-dimensional quantities

$$\left.\begin{aligned} & r^*=\frac{r'}{R_0},\mathrm{z}^*=\frac{z'}{R_0},\theta=\frac{T'-T_\infty}{T'_w-T_\infty},w^*=\frac{w'}{w_0},t^*=\frac{t'v}{R_0},P'=\frac{p^*-p_w}{\mu},\xi=\frac{R_0}{w_0}, \\ & Pr=\frac{\mu C_p}{k_T},Gr=\frac{g\beta_T R_0^3}{w_0 \nu}\left(T'_w-T_\infty\right),Da=\frac{K}{R_0^2},M=B_0R_0\sqrt{\frac{\sigma_c}{\mu_e}},Re=\frac{\rho v R_0}{\mu} \end{aligned}\right\} \tag{2.5}$$

Substitute the equation (2.5) into (2.2) and (2.3), we get dropping the asterisk

$$Re\frac{\partial w}{\partial t}=-\xi\frac{\partial p}{\partial z}+\frac{1}{1+\xi_1}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)-\left(M^2+\frac{1}{Da}\right)w+Gr\theta \tag{2.6}$$

$$PrRe\frac{\partial\theta}{\partial t}=\left(\frac{\partial\theta^2}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}\right) \tag{2.7}$$

The corresponding non-dimensional boundary conditions are

$$\begin{aligned} & w=0\ ,\ \ \theta=1 \text{ at } r=1 \\ & w=0,\ \ \theta=0 \text{ at } r=a \end{aligned} \tag{2.8}$$

3 Methods of Solution

Since the flow of blood through an artery is largely dependent on the pumping action of the heart and it gives rise to an oscillatory pressure gradient on the left ventricle which can be represented as

$$-\frac{\partial p^*}{\partial x}=P_0e^{i\omega t} \tag{2.9}$$

and define the velocity and temperature profiles as

$$w\left(r,t\right)=w_0(r)e^{i\omega t} \tag{2.10}$$

$$\theta\left(r,t\right)=\theta_0(r)e^{i\omega t} \tag{2.11}$$

where P_0 is constant pressure, ω is the angular frequency of the oscillation. Substituting the above expression in Equations (2.9), (2.10) and (2.11) into equations (2.6) and (2.7), we obtain:

$$\left(\frac{\partial^2 w_0}{\partial r^2}+\frac{1}{r}\frac{\partial w_0}{\partial r}\right)-\left(1+\xi_1\right)\left(M^2+\frac{1}{Da}+Rei\omega\right)w_0+Gr\left(1+\xi_1\right)\theta_0=-\xi\left(1+\xi_1\right)P_0 \tag{2.12}$$

$$\frac{\partial^2 \theta_0}{\partial r^2}+\frac{1}{r}\frac{\partial \theta_0}{\partial r}-PrRei\omega\theta_0=0 \tag{2.13}$$

Let $\beta^2=\left(1+\xi_1\right)\left(M^2+\frac{1}{Da}+Rei\omega\right), \lambda^2=PrRei\omega, \alpha_1=\left(1+\xi_1\right)Gr$ and $\alpha_2=\xi\left(1+\xi_1\right)$ so that

$$\frac{\partial^2 w_0}{\partial r^2}+\frac{1}{r}\frac{\partial w_0}{\partial r}-\beta^2 w_0+\alpha_1\theta_0=-\alpha_2 P_0 \tag{2.13a}$$

$$\frac{\partial^2 \theta_0}{\partial r^2}+\frac{1}{r}\frac{\partial \theta_0}{\partial r}-\lambda^2\theta_0=0 \tag{2.13b}$$

And the corresponding boundary conditions are:

$$w_0=0,\theta_0=0,r=a,\theta_0=1,r=1 \tag{2.13c}$$

Equations (2.13a) and (2.13b) are coupled non-linear differential equations and are solved analytically with the appropriate boundary condition in equation (2.13c) as

$$\theta_0(r)=A_{51}I_0(\lambda r)+A_{52}K_0(\lambda r) \tag{2.14}$$

$$A_{51}=\frac{K_0(\lambda \mathrm{a})}{I_0(\lambda)K_0(\lambda \mathrm{a})-K_0(\lambda)I_0(\lambda \mathrm{a})}, A_{52}=\frac{I_0(\lambda \mathrm{a})}{I_0(\lambda)K_0(\lambda \mathrm{a})-K_0(\lambda)I_0(\lambda \mathrm{a})} \tag{2.15}$$

Substituting equation (2.15) into equation (2.14), it can be rewritten as

$$\theta_0(r)=\frac{K_0(\lambda \mathrm{a})}{I_0(\lambda)K_0(\lambda \mathrm{a})-K_0(\lambda)I_0(\lambda \mathrm{a})}I_0(\lambda r)+\frac{I_0(\lambda \mathrm{a})}{I_0(\lambda)K_0(\lambda \mathrm{a})-K_0(\lambda)I_0(\lambda \mathrm{a})}K_0(\lambda r) \tag{2.16}$$

Substitute equation (2.16) into the momentum equation in (2.13a), so that we can write the aforementioned equation as

$$\frac{\partial^2 w_0}{\partial r^2}+\frac{1}{r}\frac{\partial w_0}{\partial r}-\beta^2 w_0=-\left(\alpha_2 P_0+\alpha_1\theta_0\right) \tag{2.17}$$

Solving equation (2.17) analytically using the appropriate boundary condition in equation (2.13c), we shall have the following solution in dimensionless form:

$$w_0(r)=A_{53}I_0(\beta r)+A_{54}K_0(\beta r)+\frac{\alpha_2 P_0}{\beta^2}+\frac{\alpha_1 Gr}{\beta^2}\left(A_{51}I_0(\lambda r)+A_{52}K_0(\lambda r)\right) \tag{2.18}$$

$$w_0(1) = A_{53}I_0(\beta) + A_{54}K_0(\beta) + \frac{\alpha_2 P_0}{\beta^2} + \frac{\alpha_1 Gr}{\beta^2}H(\lambda) = 0 \tag{2.19}$$

$$w_0(a) = A_{53}I_0(\beta a) + A_{54}K_0(\beta a) + \frac{\alpha_2 P_0}{\beta^2} + \frac{\alpha_1 Gr}{\beta^2}H(\lambda a) = 0 \tag{2.20}$$

where $H(\lambda) = \left(A_1 I_0(\lambda) + A_2 K_0(\lambda)\right)$

$$A_{53}I_0(\beta) + A_{54}K_0(\beta) = -\frac{1}{\beta^2}\left(\alpha_2 P_0 + \alpha_1 Gr H(\lambda)\right) \tag{2.21}$$

$$A_{53}I_0(\beta a) + A_{54}K_0(\beta a) = -\frac{1}{\beta^2}\left(\alpha_2 P_0 + \alpha_1 Gr H(\lambda a)\right) \tag{2.22}$$

Our velocity profile is obtained as:

$$w(r) = \left\{A_{53}I_0(\beta r) + A_{54}K_0(\beta r) + \frac{\alpha_2 P_0}{\beta^2} + \frac{\alpha_1 Gr}{\beta^2}\theta_0\right\}e^{i\omega t} \tag{2.23}$$

where

$$A_{53} = \frac{1}{\beta^2}\left\{\frac{\left(\alpha_2 P_0 + \alpha_1 Gr H(\lambda a)\right)}{I_0(\beta)K_0(\beta a) - I_0(\beta a)K_0(\beta)}K_0(\beta) - \frac{\left(\alpha_2 P_0 + \alpha_1 Gr H(\lambda)\right)}{I_0(\beta)K_0(\beta a) - I_0(\beta a)K_0(\beta)}K_0(\beta a)\right\} \tag{2.24}$$

$$A_{54} = \frac{1}{\beta^2}\left\{\frac{\left(\alpha_2 P_0 + \alpha_1 Gr H(\lambda)\right)}{I_0(\beta)K_0(\beta a) - I_0(\beta a)K_0(\beta)}I_0(\beta a) - \frac{\left(\alpha_2 P_0 + \alpha_1 Gr H(\lambda a)\right)}{I_0(\beta)K_0(\beta a) - I_0(\beta a)K_0(\beta)}I_0(\beta)\right\} \tag{2.25}$$

and the temperature profile as:

$$\theta(r) = \left\{\begin{array}{l}\dfrac{K_0(\lambda \mathrm{a})}{I_0(\lambda)K_0(\lambda \mathrm{a}) - K_0(\lambda)I_0(\lambda \mathrm{a})}I_0(\lambda r) + \\ \dfrac{I_0(\lambda \mathrm{a})}{I_0(\lambda)K_0(\lambda \mathrm{a}) - K_0(\lambda)I_0(\lambda \mathrm{a})}K_0(\lambda r)\end{array}\right\}e^{i\omega t} \tag{2.26}$$

4 Discussion of the Results

In this section, the numerical and computational results are discussed for the problem of an incompressible non-Newtonian Jeffrey fluid through indented arterial channel in detail with graphical illustrations. The numerical evaluations of the analytical results and some important results are displayed graphically in Figs. 1 to 10. MATHEMATICA is used to obtain the numerical results and illustrations. The analytical solution of the momentum equation is obtained by Frobenius method. All the obtained solutions are discussed graphically under the variations of various important parameters in the present section.

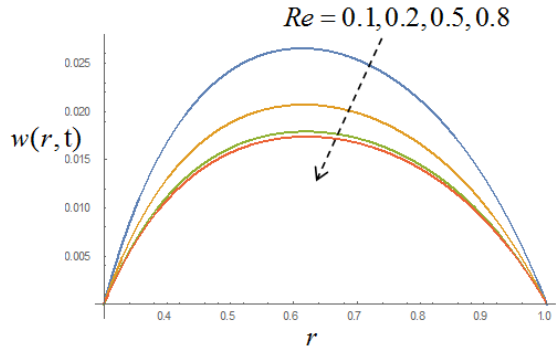


Fig. 1. Velocity Profile $w(r,t)$ **against** r **with variation of** Re **, leaving** $a = 0.3, \xi = 0.5, Gr = 0.1,$ $Da = 0.05,\ \omega = 0.5, Pr = 21,\ M = 0.5,\ \xi_1 = 0.2$ **.We observed that the velocity decreases as** Re **increases**

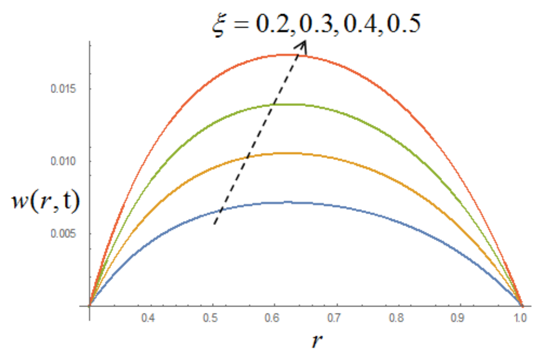


Fig. 2. Velocity Profile $w(r,t)$ **against** r **with variation of** ξ **, leaving** $a = 0.3, Gr = 0.1, \Pr = 21,$ $Da = 0.05\ \omega = 0.2,\ M = 0.5, t = 0.3, Re = 0.7,\ \xi_1 = 0.2$ **We observed that the velocity increases as the relaxation** ξ **increases**

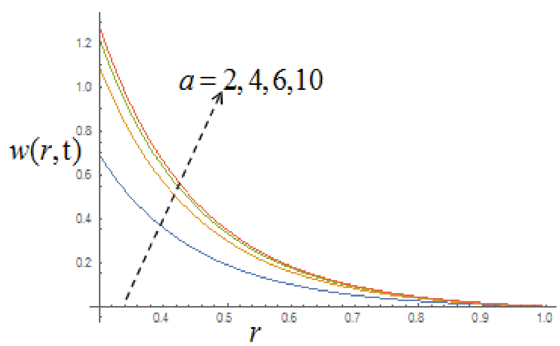


Fig. 3. Velocity Profile $w(r,t)$ **against** r **with variation of** a **, leaving** $\xi = 0.5, Gr = 0.1,\ Da = 0.05$ $\omega = 0.2,\ M = 0.5, t = 0.3, Re = 0.1, \Pr = 21,\ \xi_1 = 0.2$ **constant. It is observed that the velocity decreases as the stenotic region** a **increases**

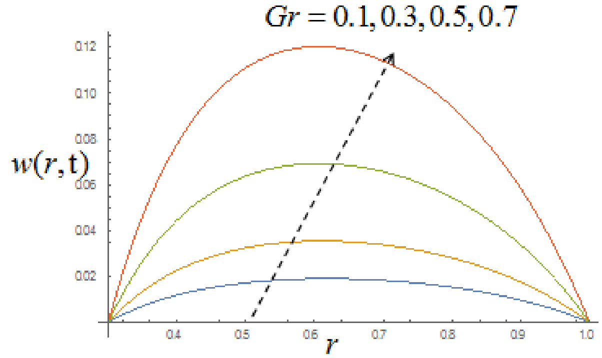


Fig. 4. Velocity Profile $w(r,t)$ **against** r **with variation of** Gr **, leaving** $a = 0.3, \xi = 0.3,\ Da = 0.05$ $\omega = 0.2,\ \mathrm{M} = 0.5, t = 0.3, Re = 0.5, \Pr = 21,\ \xi_1 = 0.2$ **constant. It is observed that the velocity increases as the Grasof number,** Gr **increases**

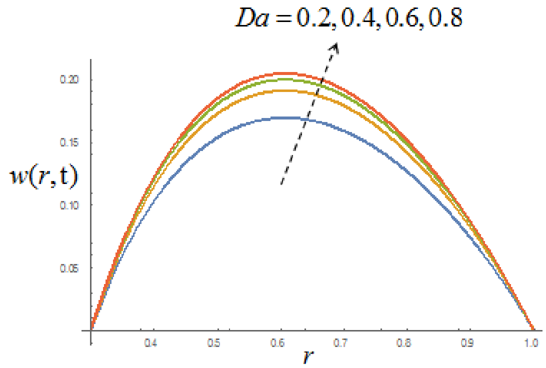


Fig. 5. Velocity Profile $w(r,t)$ **against** r **with variation of** Da **,** $a = 0.3, \xi = 0.5, Gr = 0.7,$ $Re = 0.5\ \omega = 0.02, Pr = 21,\ \mathrm{M} = 2,\ \xi_1 = 0.2$ **constant. We observed that the velocity increases as** Da **number increases**

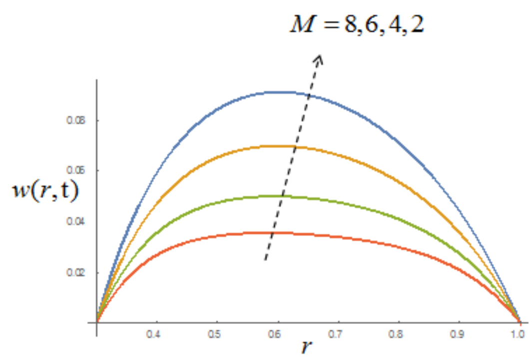


Fig. 6. Velocity Profile $w(r,t)$ **against** r **with variation of** M **, leaving** $a = 0.3, \xi = 0.5,\ Da = 0.05,$ $\omega = 0.05, Pr = 21,\ Re = 0.5\ Gr = 0.7,\ \xi_1 = 0.2$ **constant. It is observed that the velocity increases as the magnetic field** M **decreases**

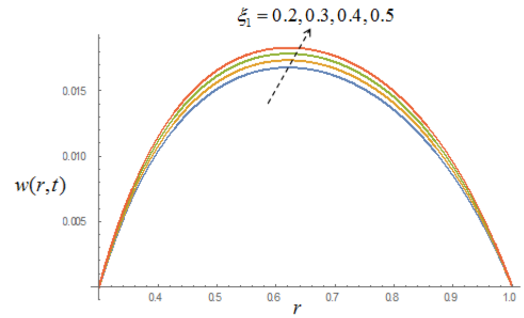


Fig. 7. Velocity Profile $w(r,t)$ against r with variation of ξ_1, leaving $Gr = 0.1$ $a = 0.3$, $Re = 0.7$, $Da = 0.05$, $M = 0.5$ $\xi = 0.5$ $\omega = 0.02$, $Pr = 21$ $t = 3$, remain constant. We observed that the velocity profile increases as ξ_1 increases

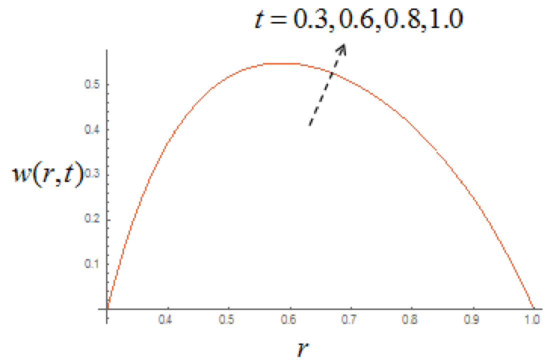


Fig. 8. Velocity Profile $w(r,t)$ against r with variation of t, leaving $a = 0.3$, $Re = 0.2$, $\omega = 0.01$, $\xi_1 = 0.2, \xi = 0.5, Gr = 0.1, Da = 0.05$ $Pr = 21$ remain constant. It is observed that the temperature profile $w(r,t)$ does not change as t increases

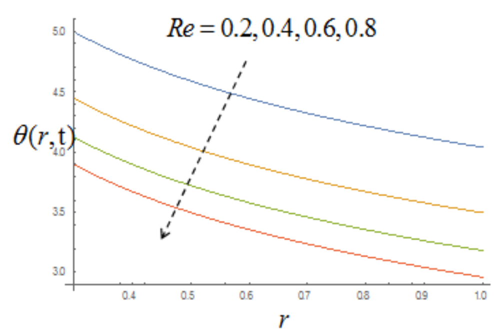


Fig. 9. Temperature Profile $\theta(r,t)$ against r with variation of leaving $a = 0.3$, $Pr = 21$ $\omega = 0.2$, $t = 0.5$ remain constant. It is observed that the temperature profile $\theta(r,t)$ decreases as Re increases

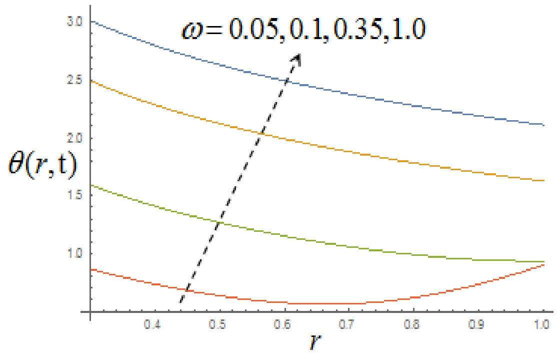


Fig. 10. Temperature Profile $\theta(r,t)$ against r with variation of leaving $a = 0.3,\ Re = 0.2,\ Pr = 21$ $t = 0.5$ remain constant. It is observed that the temperature profile $\theta(r,t)$ decreases as ω increases

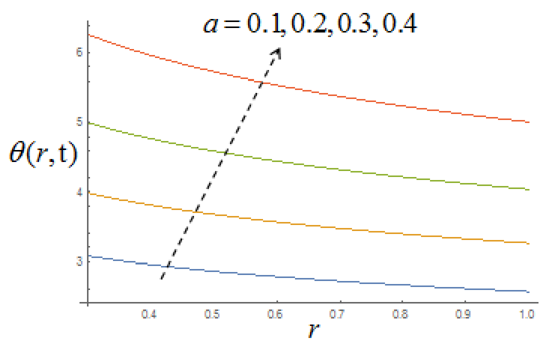


Fig. 11. Temperature Profile $\theta(r,t)$ against r with variation of leaving $Pr = 21$ $Re = 0.2,$ $\omega = 0.2,\ t = 0.5$ remain constant. It is observed that the temperature profile $\theta(r,t)$ increases as a increases

The effect of various parameters on the flow profile and temperature profile were discussed through Fig. 1 – Fig. 10. In Fig. 1 it is observed that the velocity profile is been affected by the increase in Re by the decreasing the velocity. It also shows a parabolic shape an attained a maximum velocity at the centre. From Fig. 2 it is noticed that the velocity is caused to increase for increasing constant parameter ξ. In Fig. 4 we can observe that the increase in Grashof number Gr caused an increase in the velocity which means it is good control of blood pressure in cardiovascular challenges. Similarly, in Fig. 3 it is observed that as the velocity of the flow is increasing as the geometry of the stenosis, $a = \dfrac{R}{R_0}$ is increasing, which means from the treatment perspective it is a welcome development. Because $R \ll R_0$ and the height of the stenosis $\delta \ll 1$ maintaining the other parameters values. It is observed from Fig. 5, that the velocity profile increases as the Darcy number increases. As a matter of fact that would keep the hemoglobin in microcirculation to keep lives in check. From Fig. 6, it is observed that the velocity profile is caused to increase by reducing the amount of magnetic intensity. The velocity profiles are parabolic in nature, in which it attains it maximum at the interface. Fig. 7 is showed to study the effect of Jeffrey parameter on the flow pattern. It is observed that the velocity increases with increasing Jeffrey parameter ξ_1. And also, the velocity for any given ξ_1 is a parabolic.

5 Conclusion

In this research, we studied the effects of magnetic field and heat transfer on an oscillatory flow of Jeffrey fluid through a porous and indented artery. The expressions for velocity and temperature profiles are obtained analytically and plots were obtained using Mathematica program. The results are analyzed for different values of the important parameters namely Re , Reynolds number, ξ_1 , Jeffrey parameter, oscillatory frequency ω, Darcy number Da, Magnetic parameter M , Prandtl number for blood Pr , and radius of stenosis a .

Thus, blood velocity can be controlled by suitably adjusting (increasing/decreasing) the magnetic field strength/the slip coefficient and other physical parameters. The results presented should be of sufficient interest to surgeons who usually want to keep the blood flow rate at a desired level during the entire surgical procedure.

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Competing Interests

Authors have declared that no competing interests exist.

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