

On the Bernoulli Equation in the Free Atmosphere and Mechanism of Long-lived Vortex Formation

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

A new universal mechanism of the formation of long-lived atmospheric vortex is proposed. It is based on the hydrodynamic pressure drop in the air stream, if air density decreases in the flow direction. The magnitude of this drop may increase with jet velocity increasing. It is assumed that jet, linking the atmospheric layers with different air density, may have a local area of low pressure which can generate the vortex. The possible conditions for the emergence and intensification of atmospheric vortices of various types are discussed.

Keywords: Atmospheric vortex; hurricane; tornado; vortex formation mechanism.

1. INTRODUCTION

Atmospheric vortex as a natural phenomenon has long been known. The most formidable of its species – such as tornadoes and hurricanes – is familiar to many inhabitants of our planet as they bring them misery and troubles. Modern methods of photo- and video-fixing contain a tremendous

amount of information about atmospheric vortices from the small and harmless dust devils to deadly tornado, destroying everything in its path, and hurricanes flooding by rain vast territory. A lot of scientific papers in the field of meteorology and atmospheric physics is dedicated to the vortices, and empirical regularities of their behavior is rather well

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established and described [1,2]. As for theoretical works, the most cited ones [3-5] compare the atmospheric vortex to the Carnot heat engine, deriving the condition of its formation from thermodynamic calculations and integral laws of conservation. Meanwhile, the atmospheric vortex represents an essentially non-closed hydrodynamic system to which the conservation laws do not always apply. In any case, the physically clear and simple mechanism for the formation of atmospheric vortex is, unfortunately, missing.

In the previous work of the author [6] it was demonstrated that certain hydrodynamic "paradoxes" can have a simple explanation in the framework of classical hydrodynamics if correctly to apply it. Here we will try to use a similar approach to analyze the observed atmospheric phenomena that remain still mysterious.

The desire to find and understand a "universal" mechanism of the atmospheric vortex formation is strengthened when it becomes apparent that almost all natural vortices, regardless of their size, have many common properties. The main are the following:

1. The lower part of the atmospheric vortex axis has minimum surface atmospheric pressure;
2. The air in the vortex is moving upward by the spiral from the bottom to up;
3. Intensification of the vortex is accompanied by the acceleration of air rotation and increasing of the pressure drop in the center;
4. The formation of the vortex can occur in a relatively calm atmosphere.
5. Atmospheric vortex exists in the barotropic atmosphere: the air density depends on pressure and decreases with altitude.

In this paper we attempt to propose and analyze some new hydrodynamic factors that can lead to the formation of the stable atmospheric areas of low pressure giving the basis of various vortices formation.

2. A MENTAL EXPERIMENT

First, let's do a mental experiment, the actual implementation of which is possible in principle but beyond the scope of this work. Imagine the tube with the expansion and the fluid flowing through the tube (Fig.1). Fluid changes its

density from ρ_1 to ρ_2 in place of the extension: $\rho_2 < \rho_1$. Extension is selected in a special way so as to ensure the constancy of the average velocity v basing on the conditions of conservation of mass: $\rho v S = const$ where S is a section of pipe.

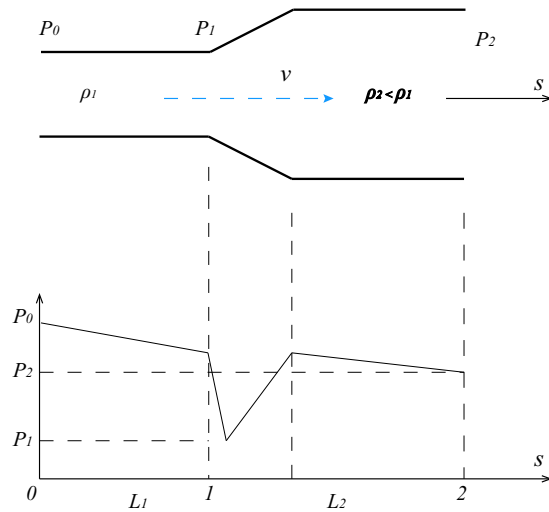


Fig. 1. Tube with extension and the fluid flow. The fluid density decreases, starting from the point 1 of the tube extension. The average flow velocity remains constant. The bottom graph shows the expected pressure distribution along the pipe

Thus, a section S of pipe, "follows" the fluid density, keeping the flow velocity constant, as $\rho S = const$.

We write the Bernoulli equation for one-dimensional flow of a fluid along a streamline with coordinate s (Fig. 1). We proceed from the generalized form of Newton's second law for a fluid with variable density (in the absence of mass forces):

$$\frac{D(\rho v)}{Dt} = -\frac{\partial p}{\partial s} - F_{fr}(v) \quad (1)$$

where p is the pressure inside the fluid, v is the linear velocity ($v = \frac{\partial s}{\partial t}$), F_{fr} is the friction force acting on a fluid particle. The pressure p_0 and p_2 on the ends of the tube is kept constant. For the steady state ($\frac{\partial(\rho v)}{\partial t} = 0$) we can write:

$$\frac{\partial(\rho v)}{\partial s} v = -\frac{\partial p}{\partial s} - F_{fr}(v), \quad (2)$$

For the first section L_1 (Fig.1), where speed increases from 0 to v and $\rho = \rho_1 = const$, integrating (2) by s gives the standard Bernoulli equation with friction:

$$p_0 - p_1 = \frac{\rho_1 v^2}{2} + \bar{F}_{fr1}(v)L_1 \quad (3)$$

where $\bar{F}_{fr1}(v)L_1 = \int_0^1 F_{fr1}(v)ds$ is the integral of

the friction loss in the first section. Accordingly, in the second section, where the density decreases and the velocity remains constant, we have:

$$p_1 - p_2 = (\rho_2 - \rho_1)v^2 + \bar{F}_{fr2}(v)L_2 \quad (4)$$

It follows immediately from (4) that there will be no pressure deficit ($p_1 \geq p_2$), if the friction in the second section is sufficiently large. From equations (3) and (4) we can evaluate the "bottom" of pressure drop p_1 :

$$p_1 = \frac{p_0 + p_2}{2} - \left(\frac{3\rho_1}{2} - \rho_2\right) \frac{v^2}{2} + \frac{1}{2}(\bar{F}_{fr1}L_1 - \bar{F}_{fr2}L_2) \quad (5)$$

Thus, for fluid flow with variable density it is possible to expect a zone of reduced pressure in areas where the density begins to decrease. This pressure drop is greater, the greater the difference in densities and greater the speed of fluid flow.

Neglecting the forces of friction, it is possible to show that the described system has an instability with positive feedback between the flow velocity and pressure drop. Indeed, from equations (3) and (4) it follows that the increase of the pressure drop on the second section (i.e., the decreasing of pressure p_1) leads to an increase in flow velocity in the first section, in accordance with (3), and the velocity growth should decrease p_1 in accordance with (4), etc. An obvious necessary condition for this is

$$(\rho_1 - \rho_2)v^2 > \frac{\rho_1 v^2}{2} \quad \text{or} \quad \rho_2 < \frac{\rho_1}{2} \quad (6)$$

However, the velocity will not increase indefinitely, and the pressure p_1 will not drop to zero. This will be prevented by friction which we for clarity have been neglected. Indeed, all friction forces F_{fr} increase with velocity increasing and, as follows from (4), the friction in the second section will reduce the pressure drop, which should stabilize the situation. Knowledge of the dependencies $F_{fr}(v)$ allows to solve equations (3) and (4) and to obtain a unique value of flow velocity v and pressure p_1 at given ρ_1, ρ_2 .

The criterion of instability (6) seems too simple, but in reality it needs to be corrected because it supposes constant velocity and does not include friction loss. However, a fundamental aspect of the instability mechanism does not disappear, because the pressure drop comes from a fundamental decrease in the momentum ρv because of decrease of density, and it should be compensated by the pressure forces in accordance with the equations of hydrodynamics.

3. DISCUSSIONS WITH EXTRAPOLATION IN CASE OF THE FREE ATMOSPHERE

It is well known that vertical air flows both ascending and descending are one of the main forces of the Earth weather and climate formation. Appearance of thunderstorms, squall lines and supercells is accompanied by intensive and local updraft of moist air. Weak, long and large-scale ascending air movements in clusters under certain conditions result in tropical storm and hurricanes formation.

Classical atmospheric convection results in considerable updraft velocity only in case of closed volumes of gas with density substantially smaller than the surrounding air density. It occurs, for example, in case of the stratosphere balloons filled with helium. Actually the convection of hot air demonstrates rather small values of lifting velocity as one can notice watching the camp-fire flame tongues (the temperature is higher than 1000 C) or usual water vapor (temperature 100 C) rising over a boiling pan. Obviously, there exists another mechanism creating in the bottom and middle troposphere intensive and long-living zones of air lifting with speeds exceeding units and even tens meters per second. These zones are well

familiar to passengers of the airliners getting to air-pockets, parachutists and glider pilots who have appeared near the thunderstorm cloud base, tourists and climbers on mountain slopes. In all these cases there are no appreciable horizontal temperature differences. Therefore, the mechanism of air lifting should be caused by the hydrodynamic reasons.

There is obvious but not always a realized feature of behavior of the air volumes being inside the atmosphere. The weight of these volumes is balanced by pushing-out force and at equality of temperatures they are indifferently balanced. Still Prandtl in its well-known work [7] wrote about some excess pressure appearing in fluid owing to its movement. This pressure « ... is distributed in liquid how if it was weightless and possessed only inert mass... Each particle of weighty fluid as though hangs in a flow under action of supporting forces of surrounding particles». Any slightest force applied to the volume in the vertical direction (it is unimportant – up or down) sets it in motion the same way as if this force was applied horizontally. Gravitational force is balanced by a vertical gradient of hydrostatic pressure. Therefore, for example, the balloon filled with room air can be displaced equally easily aside and to the ceiling.

Therefore, considering the movement of air directed vertically upwards, you can "forget" about the weight of the air, as it is compensated by Archimedes force, and suppose that the initial pressure at all altitudes "relatively" the same and equal to some reference. And the beginning of the movement caused by primary overpressure. For us it is important that there exists a force that can move air upward into the zone, where the air density is much less. The source of such power can be a deceleration of air flow due to various reasons [8,9]. When reducing the flow velocity to zero in the zone of collision with a stationary surface in the so-called critical point [6], the pressure increases on $\frac{\rho v^2}{2}$. This excess pressure either stops the flow or forces it to change its direction in accordance with the surrounding conditions. In a free atmosphere such a situation may arise, for example, if counter flows collide. In this case the mutual deceleration of both streams is inevitable with the formation of a zone of excess pressure which forces the air to move in the vertical direction. The collision of the counter-flows takes place in case of movement of air masses inside the zone of reduced pressure that may exist near the

Earth's surface due to hydrostatic causes. At the centripetal collision a local pressure increase may even exceed $\frac{\rho v^2}{2}$, since the relative speed of the flows is equal $2v$ and the maximum possible pressure increase in the zone of deceleration in accordance with the Bernoulli principle is equal $2\rho v^2$. So, for example, the characteristic pressure drop in the center of the tornado causes a powerful centripetal flows, which are mutually decelerated, and are easy to go up, because for them it is the only free direction, and the weight of the air, as noted above, does not prevent the updraft.

The pressure drop in the point 1 of fluid flow in our thought experiment (Fig. 1) was caused by the artificial reduction in density and a corresponding expansion of the pipe to maintain a constant flow rate. It seems that in the earth's atmosphere there exists a similar natural mechanism: it is the condensation of water vapor in the stream of rising air. Indeed, when condensation of water vapor occurs, heat is released, which is determined by the latent heat of condensation and humidity. If this heat exceeds the adiabatic cooling of air (10 degrees at 1km of the updraft), a particle of air takes excess heat ΔT , which in accordance with the equation of state of a gas reduces its density to a

value $\Delta\rho = -\frac{p\Delta T}{RT^2}$, where p is the pressure at

this altitude, T is air temperature, R is the universal gas constant. If the air jet density decreases, proportionally increasing the jet cross section (the jet expands), the jet velocity can remain constant as considered above in the thought experiment. To confirm this assumption, we can mention the well-known fact of nearly constant values of tangential speed of the air jets rising in the hurricane eye wall up to very high altitudes [10].

Thus, the jet of moist air (if it rises to a great height, and moves under forcing, and not due to buoyancy) may behave similarly to fluid flow in our expanding pipe (Fig.1), and the point 0 represents the location where the jet under the action of excess pressure turns up (p_0 - corresponds to the excess pressure), the point 1 is the level of condensation in the atmosphere where the air density decrease, point 2 – the height of maximum jet lifting. The pressure drop in accordance with our mechanism should be

expected at the level of condensation, when the density of the air in the upper part of the jet becomes less than the density of the surface air.

Thus, from a hydrodynamic point of view, the formation of atmospheric vortex can happen when conditions are created for forcing updraft of the air (jet) in the upper atmosphere where the air density is reduced in accordance with the equation of state. Condensation of water vapor leads to a further density decrease. The intensification of the vortex can be expected when the density of the rising jet air, according to (6), will decrease at least twice.

Taking the theoretical results as a benchmark, try to give the interpretation of some empirical data on real atmospheric vortices. For this we use simplified (no friction) formula (4) connecting the pressure drop Δp in a vortex with the density drop $\Delta \rho$ in the jet and its speed v :

$$\Delta p = v^2 \Delta \rho \quad (7)$$

where $\Delta \rho$ is the difference between air density at the inlet of the vortex and the place where the jet ceases to exist.

May be it's easy to explain the formation of fire tornadoes during major forest fires or volcanic eruptions. Indeed, the column of hot air with temperatures above 600 K (at this temperature the density of the air is reduced in 2 times) corresponds to the criterion (6) creating near the surface a primary hydrostatic pressure drop (albeit small), which "draws" inside the cold and dense surrounding air, simulating the conditions of our mental experiment.

The origin of dust "devil" can also be caused by the air heating. In desert areas, where "devils" are usually formed, the local heating of the surface air up to temperatures of 50-60 C is common. If air with such temperature formed layer, hanging or floating above the ground, the air density here will be less than the density of the surrounding air: $\Delta \rho = \rho \frac{\Delta T}{T} \approx 0,2 \text{ kg} / \text{m}^3$. If wind

stream with a speed of 10-20 m/s falls into this "hot" layer, the "cold" end of this stream will have in accordance with (7) the pressure drop of 0.2–0.8 hPa, which is typical for a small "devils".

In the powerful hurricanes the jets of moist air rise to a height of 10-12 km, while maintaining a

very high speed, up to 50-60 m/s. The density difference $\Delta \rho$ may be up to $0.8 \text{ kg} / \text{m}^3$. For example, hurricane Inez (1966) had the maximum registered wind speed equal to 80 m/s. Formula (7) gives us the evaluation of the eyewall pressure drop (not of the center of the hurricane eye), where an air jet begins its ascent: 51 hPa. Data closed to this value was registered by the researchers [10]

Tornado is one of the most impressive varieties of atmospheric vortex. Unlike a hurricane, it is accessible to observation with the naked eye. Pressure drop and wind speed of a powerful tornado, as a rule, exceed the same parameters for hurricanes [11]. And here, in connection with the proposed mechanism, there are new questions yet to be answered. First, the air is subjected to the ideal gas law connecting pressure of gas and its density. If an air jet connects regions with a different air density and obtains a pressure drop, then this would decrease the density of the air. So there is an additional positive feedback mechanism that can augment the process of pressure drop in the air. Perhaps this can explain the known "landing" of the tornado funnel and the spread of air rotation near surface. Secondly, the intensification of tornadoes and water spouts occurs with the air updraft inside the central core [12]. This fact agrees well with our theory because stream in the central core effectively connects the atmospheric levels with different air density. The existence of the only one hydrodynamic jet helps to explain the often observed bizarre shape of a tornado, its curved thin "trunk" touching the earth and rotating with high speed. Our mechanism does not need the vertical position of jet. It is not hydrostatic. Jet, that is the line current, may be of any shape: it is important that it had no ruptures and connected the region with high air density (the surface of the Earth) and area, where the air density is small (the upper atmosphere).

The air updraft in the central core is characteristic for the hurricane formation: they arise in clusters after a long ascent of moist air in the center of a very weak circulation (with wind speeds of about 1-2 m/s), which is then amplified. However, the rotation in a mature hurricane is integral: cumulus "towers", forming a hurricane, do not rotate, they move collectively around the hurricane eye where air movement is very weak. Maybe the uniform air stream exists in a hurricane only in the early formation stage of a tropical storm. However, the tornado has a

similar core, the space inside the funnel, which replaces the initial uniform jet. By the way, hurricanes are known to spawn tornadoes at landfall [13]. Perhaps their structure (the air ascent to the regions of the tropopause, high wind speed) facilitates the formation of areas of excessive pressure, which formed a vertically directed air jets, able to reach the upper atmosphere.

Obviously, there is no such uniform jets in the so-called "hot towers", a powerful tropical thunderstorm clouds, whose tops reach the tropopause. By the way there is no rotation. Perhaps, because this clouds are formed as a result of convection, but not by the hydrodynamic forcing.

In General, there are more questions than answers.

4. CONCLUSIONS

A new mechanism of pressure drop in the fluid stream with a density decreasing along the flow is proposed and theoretically proved. It is shown that the pressure drop in the stream is proportional to the difference of fluid densities and to the square of flow velocity. It is assumed that this mechanism is universal and may be responsible for the formation of atmospheric vortices of various types. The simplified formulas to evaluate the pressure drop in the atmospheric vortex are obtained. The conditions for the emergence and intensification of the vortex are formulated.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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