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On k-cordial Labeling of Some Graphs

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Abstract

In this paper we present k-cordiality of one point union of some path, cycle and star related graphs. We prove that bistar graph $B_{m,n}$ is k-cordial for all k, restricted square graph $B^2_{n,n}$ of Bistar is k-cordial for all k. We prove that one point union of cycle C_3 with star graph $K_{1,n}$ and the comb $P_n \bigodot K_1$ are k-cordial for all k.

Keywords: Abelian group; k-cordial labeling; bistar; comb; one point union.

2010 Mathematics Subject Classification: 05C78.

1 Introduction

Graph Theory is one of the Expanding Branches of Discrete Mathematics with applications in almost all the disciplines of Science and Technology. There are many potential fields of research in Graph Theory. Some of them are Enumeration of Graphs, Domination in Graphs, Algorithmic

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Graph Theory, Topological Graph Theory, Fuzzy Graph Theory, Labeling of Graph etc. Graph Labeling is an Integral part of Graph Theory which assigns numeral values to the vertices or edges or both, subject to certain conditions. The concept of Graph Labeling was introduced by A. Rosa in 1967. The present authors are motivated by the Research Article "A-cordial graphs" by Hovey [1].

Tao [2] and Youssef [3] have also discussed k-cordial labeling of various graphs.

Throughout this work, by a graph we mean finite, connected, undirected, simple graph $G = (V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$.

Definition 1.1. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s).

The latest updates of various graph labeling techniques can be found in Gallian [4].

According to Beineke and Hegde [5] Graph Labeling serves as a frontier between Structure of Graphs and Number Theory. For any undefined terms we relay upon Gross and Yellen [6]

Definition 1.2. Let $< A, * >$ be any Abelian group. A graph $G = (V(G), E(G))$ is said to be *A-cordial* if there is a mapping $f : V(G) \rightarrow A$ which satisfies the following two conditions when the edge $e = uv$ is labeled as $f(u) * f(v)$

(i) $|v_f(a) - v_f(b)| \leq 1$; for all $a, b \in A$,
(ii) $|e_f(a) - e_f(b)| \leq 1$; for all $a, b \in A$.

Where

$v_f(a)$=the number of vertices with label a;
$v_f(b)$=the number of vertices with label b;
$e_f(a)$=the number of edges with label a;
$e_f(b)$=the number of edges with label b.

We note that if $A =< Z_k, +_k >$, that is additive group of modulo k then the labeling is known as k-cordial labeling.

The concept of A-cordial labeling was introduced by Hovey [1].

Definition 1.3. If G is graph of order n, the corona of G with another graph H, $G \bigodot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H.

Definition 1.4. The *Bistar* $B_{m,n}$ is the graph obtained by joining the center(apex) vertices of $K_{1,m}$ and $K_{1,n}$ by an edge.

Definition 1.5. Let G be a simple connected graph. The *square of graph* G denoted by G^2 is defined to be the graph with the same vertex set as G and in which two vertices u and v are joined by an edge $\Leftrightarrow$ in G we have $1 \leq d(u, v) \leq 2$.

We note that the restricted square graph of Bistar $B^2_{m,n}$ is the graph obtained from $B_{m,n}$ by joining all the pendant vertices of the $K_{1,m}$ with the apex vertex of $K_{1,n}$ and all the pendant vertices of the $K_{1,n}$ with apex vertex of the $K_{1,m}$.

Definition 1.6. The graph obtained by joining a pendant edge at each vertex of a path P_n is called a *Comb* and is denoted by $P_n \bigodot K_1$.

Definition 1.7. The graph obtained by joining n pendant edges at one vertex of the cycle C_3 is called one point union of C_3 with $\mathrm{K}_{1,n}$.

A graph may be labeled by both the labeling techniques cordial labeling as well as k-cordial labeling at a time, may be labeled by any one of them, or may not at all be labeled by any of them. In other words these two graph labeling techniques are independent. Following are some examples to emphasize the fact.

- Fans are cordial [7] as well as k-cordial for all k [8].
- Wheels W_n are cordial if and only if $n \not\equiv 3(mod4)$ [7] where as wheels are k-cordial for all odd k if and only if $n \not\equiv \frac{k-1}{2}(modk)$.
- Cycles C_n are cordial unless C_{4n+2} [9] while cycles C_n are k-cordial for all odd k [1].

2 Main Results

Theorem 2.1. The Bistar $B_{m,n}$ is k-cordial graph for all k.

Proof. Let $G = B_{m;n}$ be the bistar with vertex set $\{u; v; u_i; v_j; 1 \leq i \leq m, 1 \leq j \leq n\}$ where u_i, v_j are pendant vertices. We note that $|V(G)| = m + n + 2$ and $|E(G)| = m + n + 1$.

Define k-cordial labeling $f : V(G) \to Z_k$ as follows.

$f(u) = k - 1$;
$f(v) = 1$;
$f(u_i) = p_i$; $(2i + 1) \equiv p_i(mod\ k)$; $1 \leq i \leq m$.
$f(v_j) = p_j$; $2(j - 1) \equiv p_j(mod\ k)$; $1 \leq j \leq n$.

The labeling pattern defined above covers all possible arrangement of vertices. In all possibilities the graph under consideration satisfies the vertex conditions and edge conditions for k-cordial labeling. Hence the Bistar $B_{m,n}$ is k-cordial for all k.

Illustration 2.2. The Bistar graph $B_{4,6}$ and its 15-cordial labeling is shown in Fig. 1.

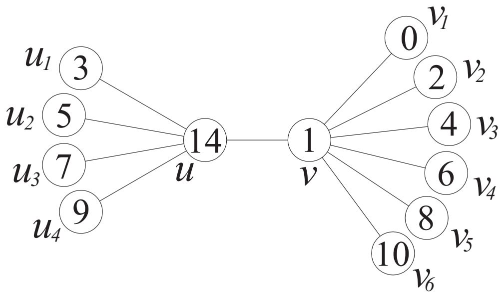

Fig. 1. **15-Cordial labeling of bistar graph** $B_{4,6}$.

Theorem 2.3. The restricted square graph $B^2_{n,n}$ of Bistar $B_{n,n}$ is k-cordial for all k.

Proof. Let $B_{n,n}$ be the bistar with vertex set $\{u; v; u_i; v_i; 1 \leq i \leq n\}$,where u_i, v_i are pendant vertices. Let G be the restricted square graph $B^2_{n,n}$ with $V(G) = V(B_{n,n})$ and $E(G)$=$\{E(B_{n,n}) \bigcup uv_i, vu_i/1 \le i \le n\}$ We note that $|V(G)|$=$2n+2$ and $|E(G)|$=$4n+1$

To define k-cordial labeling $f : V(G) \to Z_k$ we consider the following two cases.

Case 1: k is odd.

$f(u) = 0;$
$f(v) = k-1;$
$f(u_i) = 4i-1; \quad 1 \le i \le \lfloor \frac{k-1}{4} \rfloor.$
$= p_i; \quad (4i+1) \equiv p_i(mod\ k); \lfloor \frac{k-1}{4} \rfloor + 1 \le i \le n.$
$f(v_i) = 4i-3; \quad 1 \le i \le \lceil \frac{k-1}{4} \rceil.$
$= p_i; \quad (4i-1) \equiv p_i(mod\ k); \lceil \frac{k-1}{4} \rceil + 1 \le i \le n.$

Case 2: k is even.

$f(u) = 0;$
$f(v) = k-1;$
$f(u_i) = p_i; \quad (2i-1) \equiv p_i(mod\ k); 1 \le i \le n.$
$f(v_i) = k - p_i; \quad (2i) \equiv p_i(mod\ k);$ if $p_i \neq 0,$
$= 0; \quad (2i) \equiv p_i(mod\ k),$ if $p_i = 0,\ 1 \le i \le n.$

The labeling pattern defined above covers all possible arrangement of vertices. In all possibilities the graph under consideration satisfies the vertex conditions and edge conditions for k-cordial labeling. Hence the restricted square graph $B^2_{n,n}$ of Bistar $B_{n,n}$ is k-cordial for all k.

Illustration 2.4 The restricted square graph $B^2_{5,5}$ and its 15-cordial labeling is shown in Fig. 2.

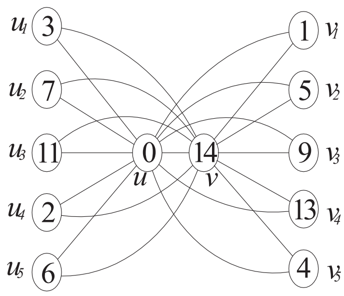

Fig. 2. **15-Cordial labeling of restricted square graph** $B^2_{5,5}$.

Theorem 2.5. The one point union of cycle C_3 with star graph $K_{1,n}$ is k-cordial for all k.

Proof. Let G be the graph obtained by one point union of cycle C_3 and star $K_{1,n}$. Let$\{u, v, w\}$ be the vertices of the cycle C_3 and $\{v_1, v_2, ..., v_n\}$ be the vertices of n pendant edges. We note that $|V(G)| = |E(G)| = n+3$.

To define k-cordial labeling $f : V(G) \to Z_k$ we consider the following two cases.

Case 1: k is odd.

$f(u) = \frac{k-1}{2};$

$f(v) = 0;$
$f(w) = \frac{k+1}{2};$
$f(v_i) = i; \qquad 1 \leq i \leq \frac{k-3}{2}$

$f(v_i) = p_i; \qquad (i+2) \equiv p_i (mod\ k); \ \frac{k-1}{2} \leq i \leq n.$

Case 2: k is even.

$f(u) = \frac{k-2}{2};$
$f(v) = 0;$
$f(w) = \frac{k+2}{2};$
$f(v_i) = i; \qquad 1 \leq i \leq \frac{k-4}{2}$

$f(v_i) = i+1; \quad i = \frac{k-2}{2}$

$f(v_i) = p_i; \qquad (i+2) \equiv p_i (mod\ k); \ \frac{k}{2} \leq i \leq n.$

The labeling pattern defined above covers all possible arrangement of vertices. In all possibilities the graph under consideration satisfies the vertex conditions and edge conditions for k-cordial labeling. Hence the one point union of cycle C_3 with star graph $K_{1,n}$ is k-cordial for all k.

Illustration 2.6 The one point union of C_3 and $K_{1,8}$ and its 11-cordial labeling is shown in Fig. 3.

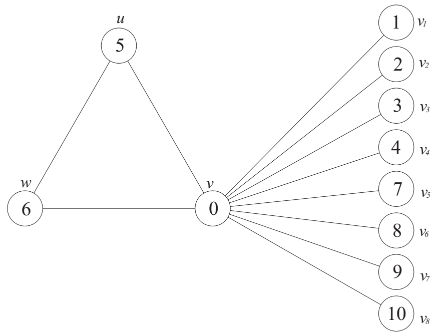

Fig. 3. **11-Cordial labeling of the one point union of C_3 and $K_{1,8}$ graph.**

Theorem 2.7. The Comb Graph $P_n \bigodot K_1$ is k-cordial for all k.

Proof. Let $G = P_n \bigodot K_1$ be the comb graph. Let $\{v_1, v_2, v_3, ..., v_n\}$ be the vertices of the path P_n and $\{u_1, u_2, u_3, ..., u_n\}$ be the vertices adjacent to each vertex of the path P_n. We note that $|V(G)| = 2n$ and $|E(G)| = 2n - 1$.

To define k-cordial labeling $f : V(G) \rightarrow Z_k$ we consider the following two cases.

Case 1: k is odd.

$f(v_i) = p_i; \qquad 2(i-1) \equiv p_i (mod\ k);\ i$ is odd,
$\qquad = p_i; \qquad (2i-1) \equiv p_i (mod\ k);\ i$ is even, $1 \leq i \leq n.$
$f(u_i) = p_i; \qquad (2i-1) \equiv p_i (mod\ k);\ i$ is odd, i is odd,
$\qquad = p_i; \qquad 2(i-1) \equiv p_i (mod\ k);\ i$ is even, $1 \leq i \leq n.$

Case 2: k is even.

Subcase I: $\frac{k}{2}$ is odd.

$$\begin{array}{ll} f(v_i) = p_i; & 2(i-1) \equiv p_i(mod\ k);\ 1 \leq i \leq n. \\ f(u_i) = p_i; & (2i-1) \equiv p_i(mod\ k);\ 1 \leq i \leq n. \end{array}$$

Subcase II: $\frac{k}{2}$ is even.

$$\begin{array}{ll} f(v_i) = p_i; & (i-1) \equiv p_i(mod\ k);\ 1 \leq i \leq n. \\ f(u_i) = p_i; & (\frac{k}{2} + (i-i)) \equiv p_i(mod\ k);\ 1 \leq i \leq n \end{array}$$

The labeling pattern defined above covers all possible arrangement of vertices. In all possibilities the graph under consideration satisfies the vertex conditions and edge conditions for k-cordial labeling. Hence the Comb Graph $P_n \bigodot K_1$ is k-cordial for all k.

Illustration 2.8 The comb graph $P_{11} \bigodot K_1$ and its 12-cordial labeling is shown in Fig. 4.

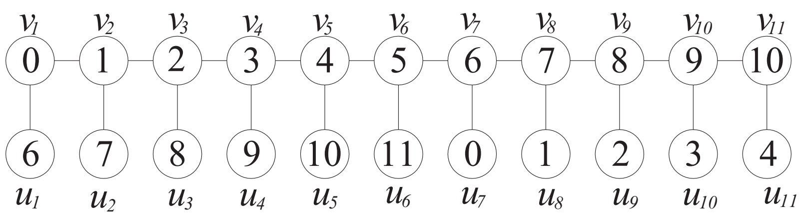

Fig. 4. **12-Cordial labeling of the comb graph** $P_{11} \bigodot K_1$.

3 Concluding Remarks

Labeled graph is the topic of current interest for many researchers as it has several diversified applications.We discuss here k-cordial labeling in the context of graph operation namely one point union. This approach is novel and contributes four new graphs to the theory of k-cordial graphs. The derived labeling pattern is demonstrated by means of sufficient illustrations which provide better understanding of the derived results. The results reported here are new and will add new dimension to the theory of k-cordial graphs.

Competing Interests

The authors declare that no competing interests exist.

References

[1] Hovey M. A-cordial graphs. Discrete Math. 1991;93:183-194.

[2] Tao R. On k-cordiality of cycles, crowns and wheels. Systems Sci. Math. Sci. 1998;11:227-229.

[3] Youssef MZ. On k-cordial labeling. Australas. J. Combin. 2009;43:31-37.

[4] Gallian JA. A dynamic survey of graph labeling. The Electronics Journal of Combinatorics. 2014;16.

[5] Beineke LW, Hegde SM. Strongly multiplicative graphs. Discuss. Math. Graph Theory. 2001;21:63-75.

[6] Gross JA, Yellen J. Handbook of graph theory CRC Press; 2004.

[7] Cahit I. On cordial and 3-equitable labellings of graphs.Util. Math. 1990;189-198.

[8] Modha MV, Kanani KK. K-cordial labeling of fan and double fan. International Journal of Applied Mathematical Research. 2015;4(2):362-369.

[9] Ho YS, Lee SM, Shee SC. Cordial labellings of unicyclic graphs and generalized Petersen graphs. Congr. Numer. 1989;68:109-122.

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