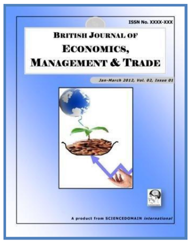

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Enhanced Index Tracking-an Extension of the Elton and Gruber (1976) Model

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Authors' contributions

Author DN designed the study, developed the models, prepared the literature review and wrote the body of the manuscript. Author BWM did the statistical analysis for the study, contributed to the revision of models, and prepared the abstract. Author TC contributed to the overall design of the research. All authors read and approved the final manuscript.

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ABSTRACT

Aims: The purpose of the study is to make a case for the development of middle-range models for use in developing markets by modifying the Elton and Gruber (1976) model to come up with semi-optimized index-tracking models with desirable tracking and excess return features.
Study Design: Non-experimental empirical design.
Place and Duration of Study: Zimbabwe, Department of Finance and Department of Insurance and Actuarial Science, covering the period between February 2009 and June 2010.
Methodology: We use weekly data of 71 industrial closing prices from the Zimbabwe Stock Exchange (ZSE) for the period starting February 2009 to June 2010 to compare the return and tracking performance of the adapted models against simple capitalization-based tracking models.
Results: We find that the semi-optimized models yield tracking and excess return results that are not statistically significantly different from simple capitalization-based models, at the 1% significance level, yet only utilizing about half as many stocks.
Conclusion: The use of semi-optimized index-tracking models has potential to significantly reduce transaction costs while keeping tracking error within reasonable limits. However, their use results in inferior excess return performance on a risk-adjusted basis

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when compared to simple capitalization-based models. The use of the correlation coefficient in filtering stocks to include in a tracking portfolio yields superior tracking error results but inferior excess return results compared to the use of the ratio of beta to idiosyncratic risk. Portfolios with higher Active Share measures produce poorer tracking error and excess return results compared to lower Active Share portfolios. The use of passive portfolio management strategies on the ZSE is supported by our findings.

Keywords: Zimbabwe; index-tracking; active share; tracking error.

1. INTRODUCTION

The decision on what assets to include in a portfolio has been the subject of academic and practical interest for ages. The theory of investment selection has evolved from simple rules such as "maximize discounted expected return" (rejected by Harry Markowitz in 1952) to today's complex portfolio optimization techniques along multiple dimensions. A notable breakthrough in investment theory was made by Harry Markowitz in 1952 with the first technical treatment of risk and return in the context of investment selection. A lucid theory of portfolio selection, now commonly referred to as Modern Portfolio Theory (MPT) was born. Markowitz's work formalized the treatment of an age-old saying "do not put all your eggs in one basket" by demonstrating how covariance in asset returns can be used to significantly reduce portfolio risk. The concept of diversification has become common wisdom in modern portfolio management practice and with the development and formalization of the efficient markets hypothesis (EMH) by [1], and subsequent development of mutual funds, investors pay a lot of attention to the extent to which their portfolios are diversified. The EMH asserts that stock prices quickly and fully incorporate all price-relevant information so that it is not possible for an investor to design an information-based trading strategy that consistently outperforms the market. While the EMH has been fiercely challenged by behavioural economists led by Robert Shiller, due to several anomalies observed in financial markets, ostensibly ascribed to investor irrationality [2], it continues to form the backbone of asset pricing and portfolio selection models [3]. Empirical evidence suggests that there is no consistent proof that markets are not efficient, especially in developed markets [3]. Most tests of the EMH that have used mutual funds and managed funds indicate that there is no consistent evidence of superiority of active management strategies over passive strategies, after adjusting for risk and transaction costs.

The evidence on market efficiency in developing markets is discouraging however, especially on African stock markets (ASMs). ASMs have been found to be inefficient even in the weakest sense [4]. Only the Johannesburg Stock Exchange (JSE) has been found to be weak-form efficient. The problem with most ASMs is illiquidity and high transaction costs. For the many small investors in developing stock markets, transaction costs per dollar invested are very high. This problem has somewhat been addressed by the introduction of unit trusts. Unit trusts are products sold by asset management companies to small investors in very small units, with each unit representing a fractional holding of the portfolio held by the trust. This way, small investors do not have to buy individual shares, which are illiquid, but instead hold units representing minute investments in several counters. This enables the investors to achieve greater diversification and also unlock liquidity at a substantially low cost. Unit trust portfolios ordinarily consist of stocks with well-defined characteristics, such as blue chip counters and growth counters. However, quite often, passive portfolios that mimic a specified benchmark index are held to meet the needs of investors. As a result, investors can access the return on the index without necessarily holding all counters in the index. Such portfolios are called tracking portfolios. This is in line with empirical work that acknowledges that high transaction costs make passive investment management more attractive than active management [5,6,7].

The classical tracking error problem focuses on minimizing the deviations from a benchmark portfolio under some restrictions. There are many different definitions of tracking error, and as a consequence, different tracking portfolio models. Tracking measures have included the correlation coefficient [8], the mean absolute deviation between portfolio and benchmark returns [9], the square root of the second moment of the deviations between portfolio returns and benchmark returns [10], and the residual volatility of the tracking portfolio with respect to the benchmark [11]. Another frequently used definition of tracking error measures the active risk of a portfolio based on the covariance matrix of the stock returns [12].

Research efforts into the index tracking problem have yielded two dominant approaches; stochastic dynamic programming [13,14,15], and heuristic algorithms [16,12,17]. The emergence of enhanced index tracking has generated a new interest in the tracking literature [10,18,19]. This has opened new lines of inquiry involving a balance between excess return and tracking error.

While a lot of attention has been paid to portfolio optimization in the literature, there is no evidence of optimizing behavior in most developing markets of the world. The lack of technological sophistication, high transaction costs, and the illiquid nature of most ASMs for example make the employment of optimization techniques a subject of pure academic debate in many cases. For a long time, trading in stock has largely involved simple rules of thumb developed over the years by market analysts, who tend to make investment decisions based on their experience with certain counters and pure gut feeling, rather than complex valuation and portfolio models. For most analysts, the susceptibility of stock trading to manipulation, the high costs associated with portfolio rebalancing, and the high sensitivity of stock markets to political sentiment discourage the use of optimized models. [20] lend support to the above by suggesting that high transaction costs may favor simple strategies ahead of optimized strategies. We note however that simple rules may not be best for tracking an index. Too many stocks may be picked for the tracking portfolio, resulting in higher transaction costs, or too few may be used, which may result in a larger tracking error than necessary. A reasonable compromise involves combining the simplicity of simple rules and the technical optimality of optimization models. The question is “how?”

Our starting point for this unconventional approach is a study of optimization models, where we seek some traces of common sense. All optimization models, whether designed for active management or index tracking, are based on some objective function and some constraints. In this paper, we focus on the objective functions. For active management, the objective is to maximize risk-adjusted returns and for passive management it is to minimize index-tracking error. Now, given that the first is a maximization problem and the second is a minimization problem, the next step would be to check for any similarities in the optimization formulae. The objective is to infuse tracking error measures into the simplest active model that resembles a corresponding tracking error minimization model; so that we achieve a reasonable trade-off between risk-adjusted returns and tracking error without the complex exercise associated with including tracking error constraints in an active model to derive a robust enhanced index-tracking model. The key here is computational simplicity! A simple, easy to understand algorithm is best. By some stroke of luck, we notice a striking resemblance between two algorithms, one developed by Elton and Gruber back in 1976 for active portfolio construction, and another developed by Glabadanidis in 2009 for index-tracking.

The general purpose of this paper is to make a case for the development of middle-range models for use in developing markets. As a first step to simplifying the process of tracking an index, this paper examines the effect of modifying the Elton and Gruber (1976) model (hereafter referred to as the E and G model), to come up with semi-optimized index-tracking models with desirable tracking and excess return features. Specifically, the study seeks to derive the best way of adapting the E and G model to index-tracking while retaining the general form of the formulae used in their active construction algorithm. The specific questions answered in this paper are as follows: Firstly, how best can the E and G model be modified to yield good tracking error results while not significantly compromising return? Secondly, what improvements do semi-optimized models make on simple tracking models? Thirdly, is there a direct relationship between the Active Share of a portfolio and tracking error? The guiding hypothesis of the study is that the use of semi-optimized models should significantly reduce the number of stocks required to achieve the same tracking results as simple models, or even better. Thus, all else equal, the use of semi-optimized index-tracking models should significantly reduce the cost of tracking an index.

This paper contributes to the literature by exploring the possibilities of achieving reasonable tracking results using remarkably simple models, based on already established optimization models. By applying the semi-optimized models to empirical data and comparing the results with results of simple capitalization-based models, the study further sheds light on the tracking features of models with varying levels of sophistication and provides further tests of claims made in the literature regarding desirable attributes of candidate stocks for a tracking portfolio. We further generate evidence on the portfolio “Active Share” measure and how it is related to different tracking error measures, as well as risk-adjusted returns. The study is an interesting contribution to the growing literature on enhanced index tracking, establishing a bridge between active and passive portfolio management strategies in a way that demands minimum computational energy. Our paper departs from mainstream investment models built for strict optimization by building simpler versions of enhanced index tracking models for an audience that is traditionally fond of “simple rules of thumb”. The good news is that there is hope that analysts in developing markets can still minimize the cost of building index trackers while avoiding the headaches of stochastic dynamic programming! We however acknowledge the limitation imposed by this simplicity; the approach may lack taste for the quants, but we know too well that all complex optimization models are white elephants in chaotic markets exemplified by most ASMs! Making the best use of simplified models makes more sense than not using any model at all.

2. METHODOLOGY

The data used in this research comprises a time series of weekly returns of all 71 industrial counters on the Zimbabwe Stock Exchange as well as corresponding ZSE Industrial index returns over a period of 78 weeks between February 2009 and June 2010. The period of the study is chosen because data prior to the year 2009 is distorted by hyper-inflation. Zimbabwe also demonetized the Zimbabwean dollar in 2009 and adopted the United States dollar (USD), among other currencies, as legal tender. Thus, we use data after the ZSE began trading in USD.

2.1 Model Development Framework

We develop and apply 4 index tracking models to the empirical data over 78 weeks. Two of the models are variants of the E and G model with a short-selling constraint. The E and G model is chosen as a platform model on account of its relative simplicity and intuitive clarity.

Furthermore, the algorithmic approach is fast to yield results without significant computing. We are further encouraged by the similarity in formulae used in an algorithm recently developed by [16] and those used in the E & G model. This is notwithstanding the fact that the E & G model was developed for active construction based on excess return optimization while Glabadanidis' algorithm was developed for index tracking.

The other 2 models are simple capitalization-based models and we use them to evaluate the semi-optimized models. This follows findings by [20] that due to high costs, optimized strategies may not perform as well as simple rules in emerging markets. The findings of [20] also inspire the static approach adopted in this study, which proposes that once a tracking portfolio is established, it will be maintained until and unless there is a change in the composition of the benchmark or there is a significant market shock which changes the covariance structure of the market. A patient approach to rebalancing is also supported by the need to minimize costs. The models are outlined briefly below.

2.1.1 The E and G model with a short-selling constraint

The E and G model is an active construction model based on the maximization of excess return to volatility. Assuming a single index model, [21] developed a model that simplifies the selection of stocks that constitute an optimal portfolio via a systematic filtering process. The rules for determining which stocks are included in the optimum portfolio are as follows:

1. Rank all stocks under consideration in descending order on the basis of their excess returns to beta (ERB); where $ERB_i = \frac{\bar{R}_{i-}R_f}{\beta_i}$ and
   $\bar{R}_i$= mean return on security *i*,
   R_f= risk free rate of return, and
   β_i= beta of security *i*.

2. The optimum portfolio consists of all stocks for which ERB is greater than a particular cut-off point, C^*. The value of C^* is calculated using the characteristics of all the securities in the optimum portfolio. Designate C_j is a candidate for C^*. The value of C_j is calculated when j securities are assumed to belong to the optimum portfolio. Since securities are ranked from the one with the highest ERB to the one with the lowest ERB, if a particular security belongs in the optimum portfolio then all higher ranked securities belong to the optimum portfolio also. The procedure for computing the values of the variable Cj starts by assuming that the first security is in the optimum portfolio (*j=1*), then the first two securities *(j=2)*, the first three securities *(j=3)* and so on. For a portfolio of j securities the value of C_j is computed using the formula $C_j = \frac{\sigma_m^2 \sum_{i=1}^{j} ERB_i \left(\beta_i^2 / \sigma_{\varepsilon i}^2 \right)}{1+\sigma_m^2 \sum_{i=1}^{j} \left(\beta_i^2 / \sigma_{\varepsilon i}^2 \right)}$
   Where:
   σ_m^2= variance of the index portfolio returns,
   ERB_i= excess return to beta for security *i,*

β_i= beta of security *i,* and
$\sigma^2_{\varepsilon i}$= unsystematic risk of security *i.*

3. The optimum C_j, that is C^*, is found when all securities used in calculating C_j have ERBs above C_j and all securities not used have ERBs less than C_j.
4. The optimum portfolio weights are calculated using the formula $\omega_i = \frac{\theta_i}{\sum \theta_i}$,

Where: $\theta_i = \frac{\beta_i}{\sigma^2_{\varepsilon i}}(ERB_i - C^*)$

2.1.2 Model 1

This model is a variant of the E & G model outlined above. It however uses asset return correlation with the index instead of excess return to beta to rank assets and screen for inclusion in the optimal portfolio. The motivation for this replacement is to reflect the shift from a focus on ERB under active management, towards an index tracking objective under passive management. Since the objective of a tracking model is to minimize tracking error (which can equivalently be taken as maximizing correlation with the benchmark index), it is reasonable to replace ERB with correlation coefficient, on the pretext that if we maximize correlation with the index we also minimize tracking error. Correlation coefficient ($\boldsymbol{\rho}_{im}$) is one of the index tracking error measures noted by [8]. This model is capitalization-neutral and relies exclusively on the historical correlation coefficient. The following formulae in the E & G model are modified as follows:

$$C_j = \frac{\sigma^2_m \sum_{i=1}^{j} ERB_i\left(\beta^2_i / \sigma^2_{\varepsilon i}\right)}{1+\sigma^2_m \sum_{i=1}^{j}\left(\beta^2_i / \sigma^2_{\varepsilon i}\right)} \qquad \text{(E and G model)}^1$$

From the market model, unsystematic risk,$\sigma^2_{\varepsilon i}$, may be expressed as $\sigma^2_{\varepsilon i} = \frac{(1-\rho^2_{im})}{\rho^2_{im}}\beta^2_i\sigma^2_m$ and when we substitute this expression into the equation for C_j, we get the following alternative expression:

$$C_j = \frac{\sum_{i=1}^{j} ERB_i\left(\rho^2_{im} / 1-\rho^2_{im}\right)}{1+\sum_{i=1}^{j}\left(\rho^2_{im} / 1-\rho^2_{im}\right)} = \frac{\sum_{i=1}^{j} ERB_i\gamma_i}{1+\sum_{i=1}^{j}\gamma_i}\left(\text{where } \gamma_i = \rho^2_{im} / 1-\rho^2_{im}\right) \text{(E \& G –revised)}$$

When we replace $\mathrm{ERB_i}$ with ρ_{im} we get the following expression for C_j under Model 1:

$$C_j = \frac{\sum_{i=1}^{j}\rho_{im}\gamma_i}{1+\sum_{i=1}^{j}\gamma_i} \qquad \text{(Model 1)}$$

Next, we modify the formula for θ_i used in calculating optimum portfolio weights as follows:

[1] *The i^{th} weight in Glabadanidis' algorithm is given by* $\omega_i = \frac{\beta_i}{\sigma^2_{\varepsilon i}} \times \frac{\beta_y}{\left[\left(\frac{1}{\sigma^2_m}\right)+\sum_j\left(\frac{\beta^2_j}{\sigma^2_{\varepsilon j}}\right)\right]}$

$$\theta_i = \frac{\beta_i}{\sigma_{\varepsilon i}^2}(ERB_i - C^*) \qquad \text{(E and G model)}$$

Again making the substitution $\sigma_{\varepsilon i}^2 = \frac{(1-\rho_{im}^2)}{\rho_{im}^2}\beta_i^2\sigma_m^2$, we get the alternative expression for θ_i as:

$$\theta_i = \frac{\rho_{im}^2}{\beta_i\,\sigma_m^2(1-\rho_{im}^2)}(ERB_i - C^*) = \frac{\gamma_i}{\beta_i\,\sigma_m^2}(ERB_i - C^*) \qquad \text{(E and G model-revised)}$$

Replacing $\mathrm{ERB_i}$ with ρ_{im} we get;

$$\theta_i = \frac{\gamma_i}{\beta_i\,\sigma_m^2}(\rho_{im} - C^*) \qquad \text{(Model 1)}$$

The determination of optimum portfolio weights remains the same under the two models.

$$\omega_i = \frac{\theta_i}{\sum\theta_i} \qquad \text{(E and G model) and (Model 1)}$$

Model 1 therefore tends to favor stocks that are highly positively correlated with the index regardless of their relative market capitalization.

2.1.3 Model 2

This Model is the same as Model 1 above except that it takes into account market capitalization and the ratio of market beta to the square root of the coefficient of non-determination$\left(\mathrm{i.e.}\ \frac{\beta_i}{(1-\rho_{im}^2)^{1/2}}\right)$. Before ranking stocks by any statistical characteristic, stocks are first ranked in descending order of their relative market capitalizations. The reasoning here is motivated by [22], who find that passive funds are more likely to overweight stocks with higher liquidity, larger market capitalizations and higher past performance. This tends to suggest that there is either return enhancement potential or tracking error reduction potential, or both, that comes with favoring higher capitalization stocks. However, it is important to note that neither Model 2 nor any other model considered in this study explicitly captures price momentum, which makes it quite interesting to investigate the vested ability of the models to indirectly capture momentum and render themselves candidates for enhanced index construction.

We impose a cardinality constraint of 15 on the array of stocks, effectively eliminating all but the top 15 stocks by market capitalization. This value is arbitrarily chosen but any upper bound on the number of stocks for consideration may be used. Instead of ranking the stocks by correlation coefficients however, we proceed to rank them in descending order according to the ratio of their market beta to the square root of their coefficient of non-determination$\left(\mathrm{i.e.}\ \frac{\beta_i}{(1-\rho_{im}^2)^{1/2}}\right)$. We then proceed as in Model 1 to compute C^*. However, we modify the formulae for computing the C_j values and the θ_i values as follows:

$$C_j = \frac{\sum_{i=1}^{j} \frac{\beta_i}{\left(1-\rho_{im}^2\right)^{1/2}} \left(\rho_{im}^2 / 1-\rho_{im}^2 \right)}{1+\sum_{i=1}^{j} \left(\rho_{im}^2 / 1-\rho_{im}^2 \right)} = \frac{\sum_{i=1}^{j} \frac{\beta_i \gamma_i}{\left(1-\rho_{im}^2\right)^{1/2}}}{1+\sum_{i=1}^{j} \gamma_i} ; \left(\text{where } \gamma_i = \rho_{im}^2 / 1-\rho_{im}^2 \right)$$

$$\theta_i = \frac{\rho_{im}^2}{\beta_i \, \sigma_m^2 (1-\rho_{im}^2)} \left(\frac{\beta_i}{(1-\rho_{im}^2)^{1/2}} - C^* \right) = \frac{\gamma_i}{\beta_i \, \sigma_m^2} \left(\frac{\beta_i}{(1-\rho_{im}^2)^{1/2}} - C^* \right)$$

$$\omega_i = \frac{\theta_i}{\sum \theta_i}$$

The motivation for the replacement of ERB with the ratio of market beta to the square root of the coefficient of non-determination is provided in [23], who decompose total tracking error variance thus:

$$\tau^2 = \alpha^2 + (\beta-1)^2 \mu_B^2 + 2\alpha(\beta-1)\mu_B + (\beta-1)^2 \sigma_B^2 + \sigma_\varepsilon^2$$

Where: α = uncorrelated security return;
μ_B= expected benchmark return
σ_B^2=variance of benchmark returns
σ_ε^2= variance of residual return component

They proceed to show that investors are largely worried about the last two terms and conclude that stocks with higher ratios of market beta to idiosyncratic risk are preferred candidates for a tracking portfolio. [16] uses the ratio $\frac{\beta_i}{\sigma_{\varepsilon i}}$ to characterize this measure of tracking potential. However, we develop an intuitively similar measure, $\frac{\beta_i}{\left(1-\rho_{im}^2\right)^{1/2}}$, which uses the coefficient of non-determination in the denominator in view of the fact that it is widely used throughout the optimization process. Thus, maximizing this ratio, just like the correlation coefficient, is assumed to lead to the minimization of tracking error.

2.1.4 Model 3

This is a simple model that recognizes the capitalization proportions of stocks in the index but does not consider any sensitivity statistics with the index. We use this model since the ZSE industrial index is a capitalization-weighted index. Thus, we would expect a capitalization-based model to produce reasonable tracking results. The model is based on the following formula:

$$\omega_i = \frac{\phi_i}{\sum_1^n \phi_i} \times 100\%$$

Where: $\phi_i = index\ weight\ of\ stock\ i$

The number of stocks in the tracking portfolio is limited to 10 in the capitalization-based models. Support for this cardinality constraint is found in [24], who find that diversification benefits are negligible for portfolios of more than 10 stocks. Although the traditional belief that holding more than 10 stocks results in superfluous diversification has been challenged in the literature [25,26], we support our choice of portfolio size by noting that the top 10 counters on the ZSE by market capitalization constitute about 73% of the total market capitalization for industrials. Furthermore, the cardinality constraint in the model is set lower than in Model 2 in view of the fact that there is no screening beyond the capitalization filtering stage. This cardinality constraint means that the *n* in the equation for Model 3 is set at 10.

2.1.5 Model 4

This model builds on Model 3 by introducing the correlation between stock returns and index returns in the construction of the tracking portfolio. The effect of incorporating the correlation coefficient is direct so that higher correlation stocks are weighted relatively higher than lower correlation stocks.

The formulae are as follows:

$$\omega_i = \frac{\psi_i}{\sum_1^n \psi_i} \times 100\%$$

Where: $\psi_i = \frac{\rho_{im}\phi_i}{1/n \sum_1^n \rho_{im}}$ and ϕ_i is as in Model 3

2.2 Model Application and Evaluation

We apply the 4 models outlined above to 78 weeks of stock returns from the ZSE. We initially assume the following rebalancing strategies to test for the better strategy for further model testing:

1. Static Tracking Strategy

Under this strategy, we set up optimal portfolios using ex post returns for the entire 78 weeks and keep the positions for the whole period.

2. Dynamic Tracking Strategy

This strategy involves setting up a portfolio using ex post returns for the first 26 weeks and keeping the positions for the next 26 weeks before rebalancing. The 26 week rebalancing is considered equivalent to 6 months and is chosen based on the conclusion by [20] that for costs of about 2%, rebalancing every six months produces the best results for emerging markets compared to alternative rebalancing schemes. We then apply the optimal weights under each model to the realized stock returns to generate a time series of portfolio returns over the entire 78 week period. While the core analysis is based on weekly data, monthly data are also derived from the weekly data to facilitate a parallel analysis to determine the time-consistency of the model results. Monthly data have been vastly used in previous empirical research on the subject. Moreso, monthly data tend to be more relevant from a practical fund management perspective, as funds are often assessed on a monthly basis on the short end of the internal evaluation spectrum.

2.2.1 Tracking Measures

We compute various tracking measures to facilitate a multi-dimensional tracking error analysis to take into account the relative merits of each measure as expounded in the vast index tracking literature. The following tracking measures are calculated for all the 4 portfolios under both the dynamic and static strategies:

- Correlation coefficient
- Mean Absolute Deviation
- Standard deviation of the difference between portfolio returns and index returns $\sigma(R_p - R_m)$
- Standard deviation of residuals, $\sigma_p\sqrt{(1-\rho_{im}{}^2)}$

The 4 models are evaluated for efficacy based on the above 4 tracking measures under the dynamic and static strategies. Apart from assessing the relative tracking efficiencies of the 4 models, we also evaluate the dynamic and static tracking strategies.

2.2.2 Other Portfolio Metrics

In addition to the 4 tracking measures above, we compute various risk and return metrics to deepen the evaluation. The first of the additional statistics is skewness of portfolio return distributions. This addition is inspired by growing evidence showing that investors exhibit positive skewness [27]. All else constant, these investors should prefer portfolios with a larger probability of very large payoffs.

In order to further study the nature of the portfolios constructed using the 4 models, we also compute portfolio Active Share measures as developed by [28]. Further to studying the extent of active management inherent in the tracking error models, we find it also interesting to evaluate the relationship between Active Share and tracking error by calculating the correlation coefficient between Active Share and the tracking error.

Despite the caution of [18] regarding the efficacy of the information ratio (IR) in evaluating active manager performance, we include the IR in the study to provide an indication of the extent to which the various tracking portfolios are able to generate adequate excess returns to compensate for any unsystematic risk retained. There is also literature that suggests that the ratio tends to favor managers whose portfolios have larger numbers [19], and this further necessitates the inclusion of the ratio to test the validity of the claim.

[22] show that there is a direct relationship between the ratios of market beta to idiosyncratic risk $\left(i.e. \frac{\beta_i}{\sigma_{\varepsilon i}}\right)$ of the stocks constituting a tracking portfolio and the tracking efficiency of that portfolio. As a result, we also compute this ratio in order to provide insight into the tracking potential of the various test portfolios. The final measure we compute is the Risk-Adjusted Excess Return (RAER) for all the portfolios. It is calculated as:

$$RAER = R_f + (\bar{R}_p - R_f)\left({}^{\sigma_p}/_{\sigma_B}\right)^{-1} - \bar{R}_B$$

Where R_f = Risk free rate of return

$\bar{R}_p$=Average portfolio return

σ_p/σ_B= Ratio of portfolio returns variability relative to the benchmark

$\bar{R}_B$= Average benchmark return

This measure was used by [18] to adjust portfolio excess returns to account for the additional total risk associated with the pursuit of excess returns by enhanced index funds. The measure provides additional information that may not be deciphered from the portfolio IR. It is worth noting that the whole study is based on the assumption that the United States dollar (US dollar) is the predominant currency of tender in Zimbabwe. Consequently, the risk-free rate used for the above computations is assumed to be on US dollar terms. While there are sovereign risk issues to consider, a risk free rate of 5% per annum is assumed for purposes of this study.

2.2.3 Analytical Methods

The study relies heavily on linear regression and correlation analysis to derive the market model. Apart from evaluating the 4 test portfolios relative to each other in excess return/tracking error space, some correlation analysis is performed between the assessment metrics to decipher any pairing tendencies. The analysis also extends to portfolio value analysis in order to assess the absolute dominance of portfolios in value space on the horizon, a matter that we consider to be of interest to investors. To add rigor to the analysis, an Analysis of Variance (ANOVA) is conducted on portfolio values, returns, and tracking errors to determine the significance of model effects on portfolio value performance, returns, and tracking error respectively. On the theoretical front, we conduct an analysis to ascertain the better of the two variants of the E and G model. To this end, we construct a comparative portfolio based on the E and G model and we conclude that the model that results in the best improvement in tracking error from the E and G model is the better version.

3. RESULTS AND DISCUSSION

Based on linear regression and correlation analysis we come up with the model inputs such as beta values and correlation coefficients. We plot regression residuals of portfolio returns against index returns and show that the regression residuals are homoscedastic, which validates the use of the linear regression model.

Table 3.1 and Table 3.2 below summarize the results of the models as applied to weekly data and monthly data respectively. The portfolios are numbered according to the model used to construct them so that Port 1 corresponds to Model 1 and so on.

Portfolio performance is generally evaluated based on two clusters of measures, that is, tracking measures and return/momentum measures.

3.1 Key to tracking and return metrics

The following are the key measures used in the model evaluation exercise:

ρ = correlation coefficient
σ = standard deviation of portfolio returns

TE_1= tracking error measured as the standard deviation of differences between portfolio returns and benchmark returns.
TE_2 = tracking error measured as the standard deviation of linear regression residuals of portfolio returns
MAD = mean absolute deviation of portfolio returns from benchmark returns.
β = historical beta coefficient, a measure of market risk
A/S = Active Share measure (measures the extent to which a portfolio is actively managed away from the benchmark)
SKEW = a measure of the extent to which a distribution is asymmetrical
IR = information ratio (ratio of average total tracking error to standard deviation of tracking error (TE_1)
R^2 = coefficient of determination, a measure of portfolio diversification
Ѓ = specific/diversifiable risk in a portfolio
RAER = risk-adjusted excess return on a portfolio
RARR = ratio of risk-adjusted returns to unadjusted returns

3.2 Portfolio Summaries: Simple Models vs. Semi-optimized Models

3.2.1 Weekly returns

When we use weekly return data to compare the performance of the 4 portfolios we have created using Models 1-4, we note that portfolio 3 dominates all others based on all but one measure of tracking error (see Table 3.1 below). Portfolio 4 gives the second best tracking performance and the highest correlation with the index, thus the highest R-square. This means that portfolio 4 provides the best diversification, which however comes at a cost in the form of lower return performance by the portfolio. From Table 3.1, portfolio 1 performs better than portfolio 2 based on TE_1, TE_2 and MAD, but worse in terms of correlation and return measures.

Table 3.1 Portfolio Summary-Static Replication-Wkly Returns

	ρ	TE_1	TE_2	MAD	β/TE_2	A/S	SKEW	IR	RAER
Port 1	0.8718	5.43%	4.90%	3.61%	27.93	68.62%	0.47	0.18	0.65%
Port 2	0.8771	6.27%	5.32%	3.92%	28.66	74.76%	0.05	0.19	0.70%
Port 3	0.8591	3.51%	3.46%	2.41%	26.33	27.36%	1.15	0.41	1.37%
Port 4	0.8824	3.63%	3.61%	2.51%	29.43	32.12%	0.64	0.36	1.09%

It is however interesting to note that while portfolio 3 provides the worst diversification, it is the least active as captured by the Active Share measure (A/S). This result is counter-intuitive as it would be expected that less active portfolios would provide the best diversification. As expected, the Active Share measure is higher for the adapted semi-optimized models. The general observation is that based on weekly returns, the simple capitalization based models (3 and 4) appear to dominate the optimization models on both tracking error and return performance. Their return distributions are more positively skewed, their information ratios (IR) are higher, and their risk-adjusted excess returns (RAER) are higher.

3.2.2 Monthly returns

When monthly returns are considered, Model 1 generates better tracking efficiency than the simple models, although it continues to be dominated in terms of return efficiency (see Table 3.2 below). Portfolio 1 provides the best diversification and the lowest TE_2. However, it performs the worst in terms of both IR and RAER. Portfolio 3 on the other hand has the best TE_1, the second worst TE_2, and the worst diversification. However, it has the best IR and RAER, and is the least active.

Table 3.2 Portfolio Summary: Static Replication-Monthly Returns

	ρ	TE_1	TE_2	MAD	β/TE_2	A/S	SKEW	IR	RAER
Port 1	0.9574	13.11%	7.98%	8.27%	20.68	68.62%	1.12	0.30	2.14%
Port 2	0.9013	20.63%	14.68%	15.00%	12.97	74.76%	1.13	0.32	2.98%
Port 3	0.8506	11.42%	11.22%	7.88%	10.09	27.36%	1.92	0.56	4.73%
Port 4	0.8885	11.54%	10.61%	7.39%	12.08	32.12%	1.78	0.49	3.81%

Portfolio 2 continues to be the worst except that it provides better diversification than portfolios 3 and 4, and better return performance than portfolio 1. The general observation is that when monthly data is considered, the semi-optimized models tend to exhibit better diversification and lower residual tracking error. Analysis over longer return periods confirms the improved tracking performance of the semi-optimized models. The study further reveals that more frequent rebalancing of tracking portfolios does not improve tracking efficiency in the Zimbabwean case, confirming existing empirical findings in emerging markets. On account of this, we proceed to adopt the static replication assumption in the rest of the analysis.

3.3 E and G Model vs. Models 1 & 2

The analysis performed hitherto has indicated that Model 1 gives better tracking error results than Model 2, albeit at a marginal cost in terms of an inferior information ratio and less risk-adjusted returns (see Table 3.2 above). Compared to the E and G model however, both Model 1 and Model 2 perform substantially better in terms of both tracking error and diversification (see Table 3.3 below). However, consistent with results from Table 3.2 above, Model 1 produces larger reductions in both total portfolio risk (σ) and tracking error (TE_1 and TE_2) than Model 2. Since tracking error is the most important factor here, we conclude that Model 1 is the better variant model for index tracking purposes.

Table 3.3 Elton and Gruber Model vs. Models 1 & 2

	σ	ŕ	*ρ*	R^2	TE_1	TE_2	RAER	SKEW	IR
E & G	36.23%	14.85%	0.4287	18%	33.99%	32.73%	2.64%	0.339	0.41
Mod 1	10.00%	1.04%	0.8718	76%	5.43%	4.90%	0.65%	0.471	0.18
Mod 2	11.07%	1.24%	0.8771	77%	6.27%	5.32%	0.70%	0.053	0.19
Diff 1	-72%	-93%	103%	313%	-84%	-85%	-75%	39%	-55%
Diff 2	-69%	-92%	105%	319%	-82%	-84%	-73%	-84%	-54%

3.4 Momentum Capture Analysis

We conduct a portfolio value migration analysis (PVMA) to determine the ability of the four models to capture return momentum across time. This simple analysis is adopted to determine any dominance effect in terms of portfolio value on the horizon. A notional portfolio of $1 million is constructed for each of the models and the value trajectories are drawn. A simple criterion based on value trajectory levels is adopted to test for dominance. The portfolio which produces the highest level value trajectory is determined to dominate all others in terms of momentum capture. The analysis reveals that, generally, the simple capitalization-based models tend to capture return momentum better than the semi-optimized models. However, the simple models tend to produce relatively higher volatility. A one-way ANOVA at the 1% level of significance on both tracking error and portfolio returns shows that both the tracking and return performance of semi-optimized models are not statistically significantly different from that of simple capitalization-based models.

3.5 Cross-Measure Correlation Analysis

We conduct a correlation analysis on the cluster of measures used to evaluate the performance of the models under review in order to unravel any relationships not captured in the main analysis and also test the authenticity of previous research findings in the context of the Zimbabwean market. Table 3.4 below shows the resulting correlation matrix for the metrics.

Table 3.4 Cross-Measure Correlation Analysis

	A/S	**TE_1**	**TE_2**	**ρ**	**β**	**RARR**	**IR**	**β/TE_2**	**$(1-\beta)^2$**
A/S	1.000	0.962	0.966	-0.003	0.974	-0.974	-0.986	-0.049	0.9165
TE_1	0.962	1.000	0.997	-0.168	0.955	-0.940	-0.947	-0.219	0.9794
TE_2	0.966	0.997	1.000	-0.186	0.950	-0.941	-0.962	-0.239	0.9617
ρ	-0.003	-0.168	-0.186	1.000	0.126	-0.151	0.014	0.996	-0.1279
β	0.974	0.955	0.950	0.126	1.000	-0.996	-0.962	0.073	0.9363
RARR	-0.974	-0.939	-0.941	-0.151	-0.996	1.000	0.973	-0.096	-0.9061
IR	-0.985	-0.947	-0.961	0.014	-0.962	0.973	1.000	0.069	-0.8776
β/TE_2	-0.049	-0.219	-0.239	0.996	0.073	-0.096	0.069	1.000	-0.1738
$(1-\beta)^2$	0.916	0.979	0.961	-0.128	0.936	-0.906	-0.877	-0.173	1.0000

The correlation matrix above shows a strong positive correlation between the Active Share measure and TE_1 and TE_2. This confirms the findings of [28], which reveal a positive relationship between Active Share and tracking error. The correlation coefficients for Active Share and TE_1 and Active Share and TE_2 are almost equal, indicating consistency. The results also reveal that higher Active Share generates higher beta values that deviate significantly from unity. However, there is no association between Active Share and both portfolio correlation coefficient and the ratio of market beta to residual volatility as captured by TE_2 (i.e. β/TE_2**)**. This implies that at the portfolio construction stage, the correlation coefficient and the ratio of market beta to residual volatility may not be useful to control for the Active Share of the resulting portfolio. Active Share is also shown to be very strongly negatively correlated with the information ratio and the ratio of risk-adjusted excess returns to non-adjusted excess returns (RARR). This result is somewhat inconsistent with [28] but vindicates proponents of passive management such as [6].

The matrix also depicts a very significant positive correlation between squared beta deviations from unity (i.e. $(1-\beta)^2$) and both TE_1 and TE_2. This is consistent with the tracking error decomposition by [23] which shows that large squared deviations of beta values of tracking portfolios from unity result in higher tracking error. There is however a weak negative correlation between $(1-\beta)^2$ and the correlation of a portfolio with the index, implying that the higher the correlation coefficient, the lower the squared beta deviations from unity. The ratio of market beta to idiosyncratic risk shows a slightly stronger negative association with squared beta deviations from unity, suggesting it may be a better filtering measure than correlation coefficient for controlling for squared deviations of portfolio beta from unity. Squared beta deviations from unity show strong negative correlation with excess return performance measures (RARR and IR), implying that larger squared beta deviations from unity are associated with lower excess return performance.

An interesting observation from the matrix is that tracking error measures are negatively correlated with the return measures, suggesting that low tracking error portfolios should generally show better return efficiency as well. The result tends to lend some support to the conjecture that the index itself is most likely efficient.

3.6 Summary of Tracking Portfolio Composition

In Table 3.5 below, we present the stock compositions of the four tracking portfolios as well as the benchmark index in order to derive simple recommendations on the potential stocks to consider when constructing an index tracker. The summary below suggests that the best 5 candidate stocks for an index tracker are Delta, Econet, Barclays, CBZ, and Innscor. The 5 stocks constitute about 56% of the total market capitalization.

Table 3.5 Stock Compositions of Portfolios 1-4

Counter	Index	Ave	ω(P1)	ω(P2)	ω(P3)	ω(P4)
Delta	14.75%	32.36%	50.42%	31.18%	20.03%	27.82%
Seedco	4.23%	2.28%	0.00%	0.00%	5.74%	3.37%
Old Mutual	3.33%	1.74%	0.00%	0.00%	4.52%	2.47%
PPC	3.04%	1.03%	0.00%	0.00%	4.13%	0.00%
Lafarge	2.66%	0.18%	0.00%	0.00%	0.00%	0.69%
OK	1.38%	0.99%	0.00%	3.95%	0.00%	0.00%
AICO	1.92%	7.56%	8.04%	13.68%	3.96%	4.55%
Barclays	5.60%	14.29%	16.45%	23.16%	7.61%	9.95%
CBZ	2.97%	12.1%	18.95%	20.26%	4.03%	5.12%
Econet	23.08%	14.58%	0.00%	0.00%	31.34%	26.98%
Hippo	4.19%	2.55%	0.00%	0.00%	5.69%	4.53%
Innscor	9.53%	8.4%	6.14%	0.00%	12.95%	14.52%
M & R	1.10%	1.94%	0.00%	7.77%	0.00%	0.00%
	77.78%	**100.00%**	**100.00%**	**100.00%**	**100.00%**	**100.00%**
			34.77%	27.72%	72.64%	72.26%

We note that while the 2 semi-optimized portfolios achieve tracking results not significantly different from the simple models, they only utilize about half the number of stocks used by the simple models. This could translate into significant cost savings in emerging markets where transaction costs are generally high. A further observation from the top 5 candidate stocks above is that all of them are market leaders in their respective industries, which suggests a tendency towards herd behavior on the ZSE.

4. CONCLUSION

We have evaluated the 4 models developed as part of this study on the basis of tracking error and return efficiency and shown that when returns and tracking error are considered, there are no significant model effects at the 1% level of significance. However, when portfolio values are considered, significant model effects exist, suggesting that capitalization-based models could be good candidates for capturing value momentum. We have also noted that the semi-optimized models are more active than the simple capitalization-based models. This is expected since the baseline model is an active optimization model. Semi-optimized models generate higher tracking error when short-term returns are considered, but tend to show remarkable improvement when longer returns are taken. The results also reveal that the semi-optimized models tend to generate higher portfolio betas that deviate significantly from unity, an indication of more exposure to systematic risk than simple models. This exposure to systematic risk is not met with greater return efficiency as evidenced by higher residual risk (which means higher total risk overall), leading to lower information ratios and risk-adjusted returns. However, semi-optimized models utilize fewer stocks to achieve tracking results not significantly different from simple models. We further note that the correlation coefficient is a more desirable stock attribute than the ratio of beta to idiosyncratic risk if tracking error alone is of the essence. However, the ratio of beta to idiosyncratic risk is more desirable if return efficiency is of the essence in the benchmark tracking problem. We have confirmed that there is a positive correlation between Active Share and tracking error. Furthermore, more active portfolios under-perform passive portfolios on a risk-adjusted basis. These results support the use of passive portfolio strategies on the ZSE. There is scope however for improving the tracking performance of semi-optimized and simplified heuristic models by developing a simplified version of Glabadanidis' model and adapting it to simple index tracking contexts such as that of Zimbabwe.

This study is affected by a few empirical issues that may affect the stability of the relationships derived. The study was conducted over a period when the Zimbabwean economy was emerging from an economic crisis and the markets were still finding their level. Consequently, there is potential, as is often the case, that the variance-covariance matrix for the market may exhibit some instability. Such instability affects beta values and correlation coefficients, which are inputs to the models developed herein. While adjusted betas have traditionally been used to correct this, we do not adjust regression-based betas in this study to maintain computational simplicity, which is a big part of this experiment. There is need however, for further studies into the stochastic properties of the variance-covariance matrix for the ZSE. Furthermore, studies are required to investigate the impact of market imperfections such as illiquidity on the efficiency of different index-tracking strategies.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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