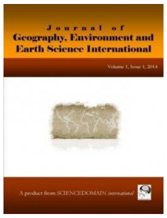

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Modeling the Dynamics of Flutes and Scallops: Preliminary Results

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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ABSTRACT

Our aim is to develop a computational model of the corrosion forms known as scallops and flutes, common in karst environment.
This model is designed to take in account dynamical interactions between successive forms, which has previously never be done, and to be as simple as possible.
We present preliminary results corresponding to numerical simulations that have been run during February 2018.
The evolution of scallops and flutes is summarized in some simple equations that model how the length of a form evolves, how too large forms split into smaller forms and how too small forms are erased by larger ones. These equations are used in programs written in C language. They enable to investigate how a system of numerous scallops or flutes evolves and to investigate the corresponding statistical distribution.
The preliminary results we present are very encouraging because of their good quantitative agreement with the well-known Curl relationship in the case of steady flows. When the velocity of the water (or air) responsible for the formation of scallops or flutes change, the model predicts that the average size of the forms changes too. However, in such a situation, the Curl relationship is not always accurately verified.

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This progressive model depends upon few parameters and does not require huge computing power. Further comparisons with field data may render it even more realistic, particularly regarding the statistical distributions it generates.

Keywords: Scallops; flutes; sauter average; curl relationship; modeling; karst.

1. INTRODUCTION

Scallops and flutes are very common in karst environments. These are corrosion forms created either when water flows in contact with soluble rocks (such as limestone or gypsum) or when air flows in contact with ice (which can sublimate) [1,2]. The formation of scallops ("scalloping") has also been reported in industrial context, for example in the power-generating industry when water flows in contact with carbon steel components [3]. However we will focus on the geological aspect of the phenomenon.

Scallops and flutes correspond to a hydrodynamical instability. Eddies forms and trigger a non-uniform corrosion (or sublimation): the resulting objects are called flutes when they look like runnels perpendicular to the mainstream and scallops when they have the shape depicted in Fig. 1. In turn, these forms maintain the generation of vorticity. For the rest of the present article, the word "scallops" will stand for both flutes and scallops unless otherwise specified.

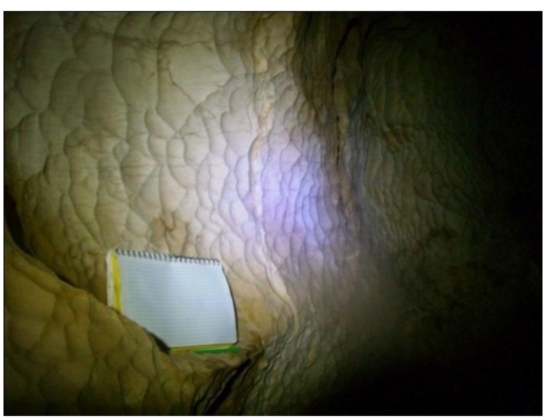

Fig. 1. Scallops inside the "Scialet de l'Aspirateur" (Vercors, France, European Union)

During the 60' and 70', Curl [4] investigated the development of scallops using analogical models made up with Paris plaster. He pointed out the importance of the turbulence and proposed a relationship between the averaged length (period) of the scallops and the velocity.

Since then, the Curl relationship is widely used in order to get information about past flows, mainly in caves (see for instance [5,6,7]). Despite this fact, several uncertainties remain regarding the detailed understanding of scallops. Different values of the Reynolds number corresponding to the Curl relationship have been proposed. For instance, whereas Curl [8] proposes R_e = (average length X velocity)/(kinematical viscosity)=*22500*, Goodchild and Ford [9] propose R_e = *11600*.

The statistical distribution of the scallops is neither symmetrical nor gaussian. Statistical distributions are given, for instance, in [9]. As explained in [2], it is widely admitted that a Sauter mean has to be done to suppress the influence of very short scallops that would occur because of bedrock heterogeneities.

Attempts have been made to model the fashion scallop forms and remain stable. Whereas certain authors are looking for very general mechanisms, the results of other authors are very dependent on precise numerical details such as profiles or solubility rates.

On the one hand, Thomas [10] puts the emphasis on eddy patterns, regardless of material removal (by dissolution or mechanic transportation) like in Fig. 2. We also have proposed in our ancient work [11] a simple link between the existence of stable scallops and the stability of the vortices (eddies) existing inside.

On the other hand, despite their good qualitative accordance, certain results of the numerical simulations carried out by Hammer, Lauritzen and Jamtveit [12] differ according to the material (limestone or plaster) and the size of the computation grid. The computational model more recently developed by Grm, Šuštar, Rodič and Gabrovšek [13] is very precisely tuned regarding dissolution rates and turbulence. This model predicts that when the initial profile isn't deep enough, no scallop will form. This seems in contradiction with the linear analysis developed by Claudin, Durán and Andreotti [14]. These models could be confronted with the experimental work of Villien, Zheng and Lister [3], which shows that the formation of scallops depends upon the presence of heterogeneities in the material ("Defect theory" versus "passive-bed theory").

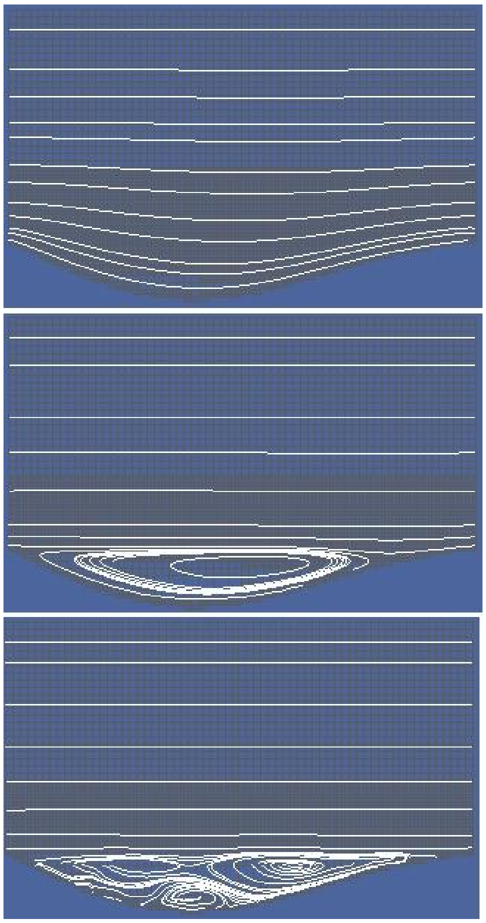

Fig. 2. When the velocity increases, an eddy appears inside the scallop. With a higher velocity, it becomes unstable
Numerical results of the author using the Flow Solver Gerris
Available:http://gfs.sourceforge.net/wiki/index.php/Main_Page
Upstream at left

Whatever the answers, the previous questions are on the subject of scallops formation and on the subject of the stability of a single scallop (or a set of identical scallops) of given profile subjected to a given velocity. Few investigation has been done about the evolution of several different scallops once formed. In their recent analogical modelling, Slabe, Hada and Knez [15] observe the formation of scallops under certain circumstances. However, this is a final observation requiring a partial destruction of the experimental device. They do not investigate the way scallops move once formed, or interact with each other. Even in experimental conditions, with a faster pace than in natural conditions, such a dynamical observation remains difficult.

This is why we are developing a dynamical model, using the simplest possible elements, in the way of [16]. For the first time (according to the literature review we have made) we are able, although roughly, to model a system of numerous different scallops interacting with each other. This theoretical and computational approach is interesting because it reconciles facts that, before, would have been regarded as contradictory. On one hand, any individual scallop may be stable within a large interval of velocity but on the other hand, the way scallops evolve and interact explains the size selection corresponding to the Curl relationship. On one hand, too short scallops could disappear and too long scallops could split into shorter forms but on the other hand, a statistical equilibrium exists, corresponding to a non-gaussian distribution.

2. MATERIALS AND METHODS

2.1 The Base of the Model

It is a one-dimensional model. It can be regarded either as a model of flutes or as a simplification of the dynamics of scallops (evolving on a two-dimensional surface). A population of N scallops is studied. From upstream to downstream, the scallop (i) follows the scallop (i-1) and precedes the scallop (i+1). V stands for the velocity (water or air) and η for its kinematic viscosity. The scallop (i) has a length L_i. If scallops were stable only for a given very precise Reynolds number, they would certainly not form in natural conditions, because of velocity fluctuations (at least seasonal). Some make the hypothesis (H1) that scallops are stable within a range of Reynolds numbers: $R_{e\ min} < VL_i / \eta < R_{e\ max}$. As the sizes of scallops observed in natural conditions do not range over several decades (see Figure 19 of [1]), $R_{e\ min}$ may be close to *3400* and $R_{e\ max}$ close to *34000*, with Curl's *22500* in-between. Choosing other values, in a larger or shorter range, would give different numerical results but would not alter the essence of the model.

We make the simplifying assumption (H2) that when a scallop evolves, the material is removed only in the front part of the scallop (Fig. 3). H2 is sustained by the results of Grm, Šuštar, Rodič and Gabrovšek (see Figure 10 of [13]). Because of H2, the front edge of (i) moves only because L_i evolves, and the rear edge of (i) moves only because L_{i-1} evolves.

The eddies are localised inside the scallops (Fig. 2), thus it is reasonable to assume (H3) that the evolution of L_i depends only of (i).

Therefore, we model the evolution of L_i as follows:

(H4): If $VL_i / \eta > R_{e\ max}$ then (i) splits into two smaller scallops of lengths rL_i (upstream) and $(1-r)L_i$ (downstream). The dimensionless parameter r is within the range [0,1]. r may be close to 0.4 (Fig. 4) although choosing other values would not change the nature of the model.

(H5): If $VL_i / \eta < R_{e\ min}$ then (i) no longer evolves; its front edge no longer moves. This is a rough modelling of the fact the eddy inside the scallop disappears or becomes too weak to enhance matter transfer. Such a short scallop will be sooner or latter erased by its neighbours.

(H6): If $R_{e\ min} < VL_i / \eta < R_{e\ max}$ then L_i evolves and the front edge of (i) moves. dL_i/dt has the dimension of a velocity. The only other parameters having the dimension of a velocity are V and η / L_i. On the basis of elementary dimensional analysis, they can be combined in a lot of different ways. The simplest, that will be used in the model and that needs only one additional dimensionless parameter, is $dL_i/dt = A\, V^{\alpha}(\eta / L_i)^{1-\alpha}$.

Taking in account other quantities, such as diffusion coefficients or thermal diffusivity, that have all the same dimension than η, would simply lead to replace η by an expression such as $\eta^{\beta}\,(\text{Diffusivity})^{1-\beta}$ and dL_i/dt would remain proportional to $V^{\alpha}L_i^{\alpha-1}$. In addition, the dimensionless quantity A is not essential to the model: it can be reduced to the unity choosing an adequate time scale. There is no particular constraint on the exponent α. However, it is reasonable to assume that the larger scallops (with a more efficient eddy transferring more matter) will move the faster: values of α below 1 are unlikely.

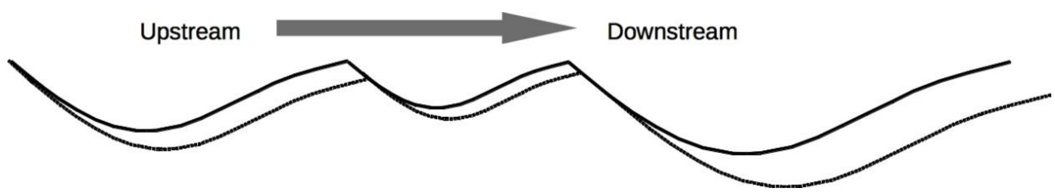

Fig. 3. Evolution of scallops
The departure of matter occurs mainly on the front part of the form
Solid line: before evolution
Dashed line: after evolution

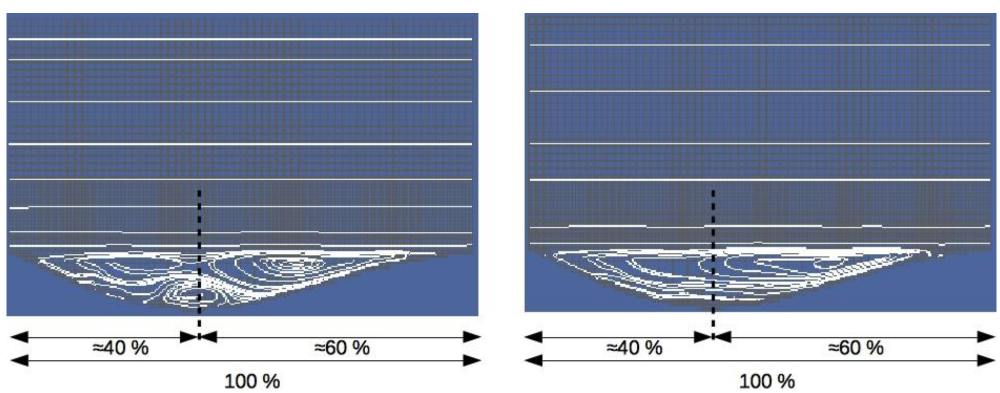

Fig. 4. Modelling the splitting of a large scallop
Numerical results of the author using the Flow Solver Gerris
Gerris is available at: http://gfs.sourceforge.net/wiki/index.php/Main_Page
Right: just before the threshold
Left: just after the threshold

2.2 The Numerical Simulation

Because of H1 to H6, it is a chaotic system the number whose degrees of freedom varies according to time. This is why, instead of focusing of the precise evolution of the lengths of a small number of scallops, which would be possible using H6, we focus on extensive numerical computation. The model is implemented using the C language in a UNIX environment. The L_i are stored in an array, whose size is dynamically adjusted when necessary. Periodic boundary conditions have been chosen in order to avoid that the number of scallops indefinitely increases. This means that the length of the last scallop has an influence on L_0. The total length of the ring of scallops remains constant during its evolution: this is a conservative system, within the meaning of [16].

The system can be initialized in several different ways: a) each scallop has a length randomly chosen within the stability interval; b) each scallop has a length randomly chosen within the stability interval but with a logarithmic the distribution; c) the scallops are randomly distributed between *90%* and *100%* of the maximal size; d) the scallops are randomly distributed between *100%* and *110%* of the minimal size; e) a scallop of maximal size is created among N-1 scallops of minimal size.

Once the system initialized, the computation can be run until a steady state is reached. Then, if necessary, the velocity can be changed in order to investigate what happens. Inside the program, the velocity is expressed in cm/s whereas the lengths are expressed in mm. In order to produce the preliminary results that will be presented and discussed below, we chose α=1, η $R_{e\,max}$ = *0.05* $m^2.s^{-1}$ and η $R_{e\ max}$ = *0.005* $m^2.s^{-1}$, η corresponding to the kinematic viscosity of water at *0* °C ($1.70\ 10^{-6}$ $m^2.s^{-1}$). Others values may be tested in further studies in order to approach the reality closer.

The “computational time” of the program is only proportional to the real time: the aforementioned quantity A has been absorbed inside. However, this simplification does not prevent relative comparisons. Anyway, enough uncertainty remains about the dissolution rates of limestone (explained for example in [17]) to prevent from using a precise value of A.

The source code has been compiled, and the programs have been run, on a laptop with a Pentium i5 processor and 8 GO RAM. The computation times were always short, by far less than a minute. The produced data have been post-processed using LibreOffice's spreadsheet.

3. RESULTS AND DISCUSSION

3.1 Steady State and Influence of the Initial Conditions

Four numerical simulations have been done with *r=0.4*, a velocity of *8 cm/s* and different initial conditions with *N=1000* scallops. Except for the very particular case of one scallop having the maximal size and *999* other having the minimal size, a steady state is quickly reached (Fig. 5).

The Sauter length corresponding to the steady state is close to *480* mm, which corresponds, using the Curl relationship, precisely to a velocity of *8* cm/s for 0°C water.

The Sauter average fluctuates. This is normal with a system of finite size. It must be pointed out that different initial conditions are tantamount to rings of different lengths, hence systems of different sizes once the steady state reached.

The final statistical repartitions of the lengths (Fig. 6) seem very few dependent on the initial conditions, except in the particular case aforementioned. There is a gap between the upper classes corresponding to very large scallops and the lower classes corresponding to small scallops. This is perfectly understandable: a scallop of intermediate size will grow if surrounded by smaller scallops, will be erased if surrounded by larger scallops. With sizes varying over a range of less than a decade, using logarithmic coordinates is meaningless. However, such a trend recalls “bimodal repartitions” evoked – because of other reasons – by Lauritzen in [1].

3.2 Length Distribution and Influence of r

Four numerical simulations have been done with r= *0.2* or *0.4* or *0.6* or *0.8* (Fig. 7), always with a velocity of *8* cm/s. The systems have been initialized with scallops of random lengths.

The Sauter lengths corresponding to the steady states are only slightly different, as well as their statistical repartitions. There is always a gap between very large scallops and small scallops. As evoked above, modifying r doesn't alter the nature of the model.

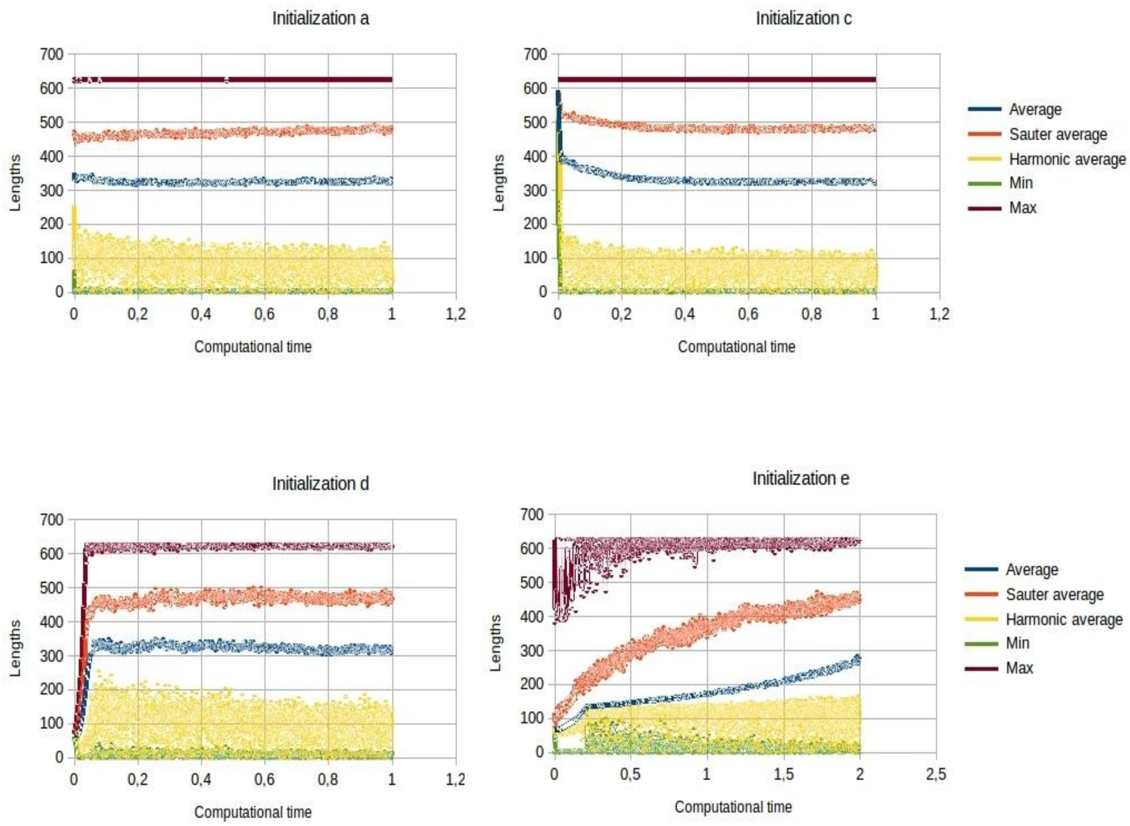

Fig. 5. Evolution of the sizes of scallops with different initial conditions

The ring initially contains 1000 scallops
Upper left: random initialization
Upper right: initialization with large scallops
Lower left: particular case, initialization with one large scallop and 999 small ones
Lower right initialization with small scallops

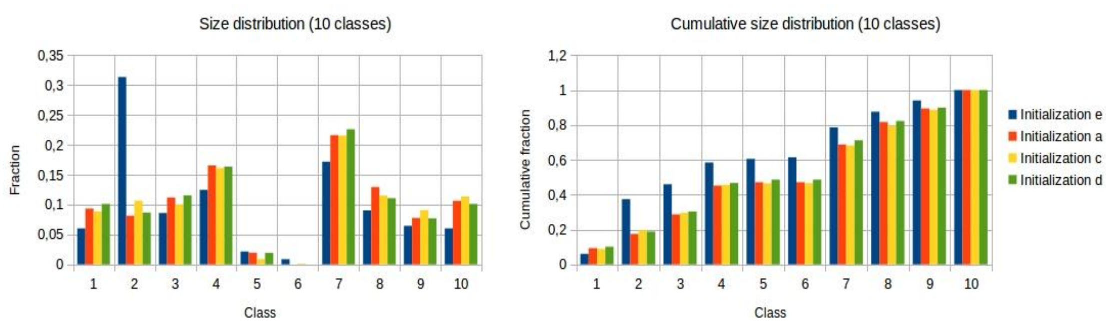

Fig. 6. Statistical distribution of the sizes of the scallops

Left: fractions corresponding to 10 classes (linear, from zero to the maximal length)
Right: corresponding cumulative distribution

The less a scallop splits in a symmetric way, the larger the gap is. The observation of natural scallops does not reveal such a marked gap between small and very large scallops. Unless observational biases could explain that, this gap must be regarded as a discrepancy between a one-dimensional model and the two-dimensional reality. Data about flutes and guided scallops (as described in [18]) could enable a deeper investigation.

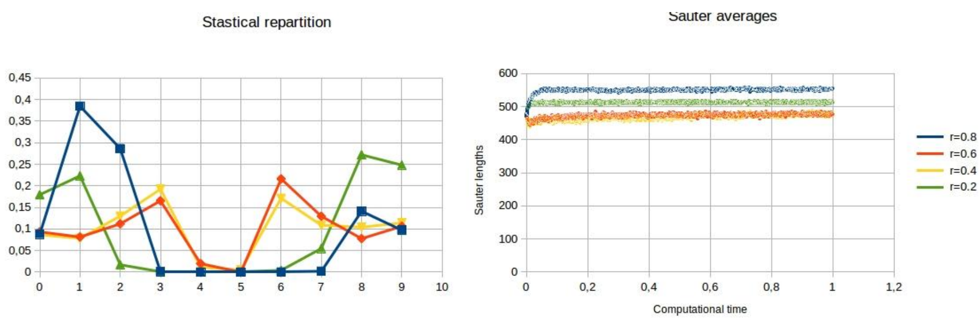

Fig. 7. Influence of r upon the statistical distribution of scallops
Left: distribution, exhibiting a gap whatever the value or r
Right: Sauter average, slightly varying with r

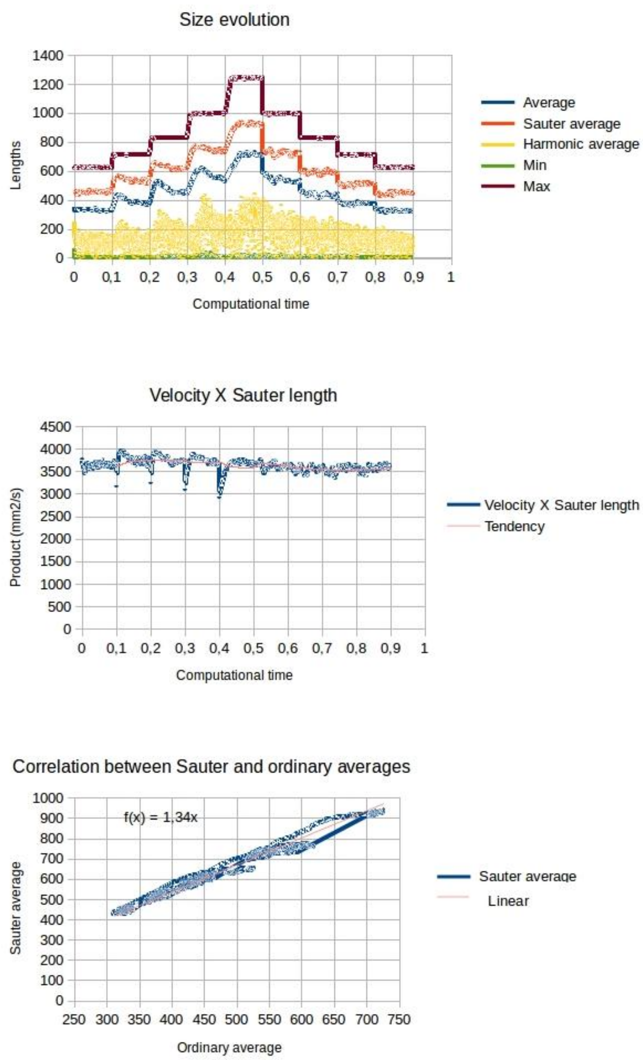

Fig. 8. Evolution of the sizes of scallops when the velocity decreases, then increases
Up: evolution of the mean lengths
Middle: fluctuations of the product velocity X Sauter length used in the Curl relationship
Down: correlations between Sauter and ordinary average

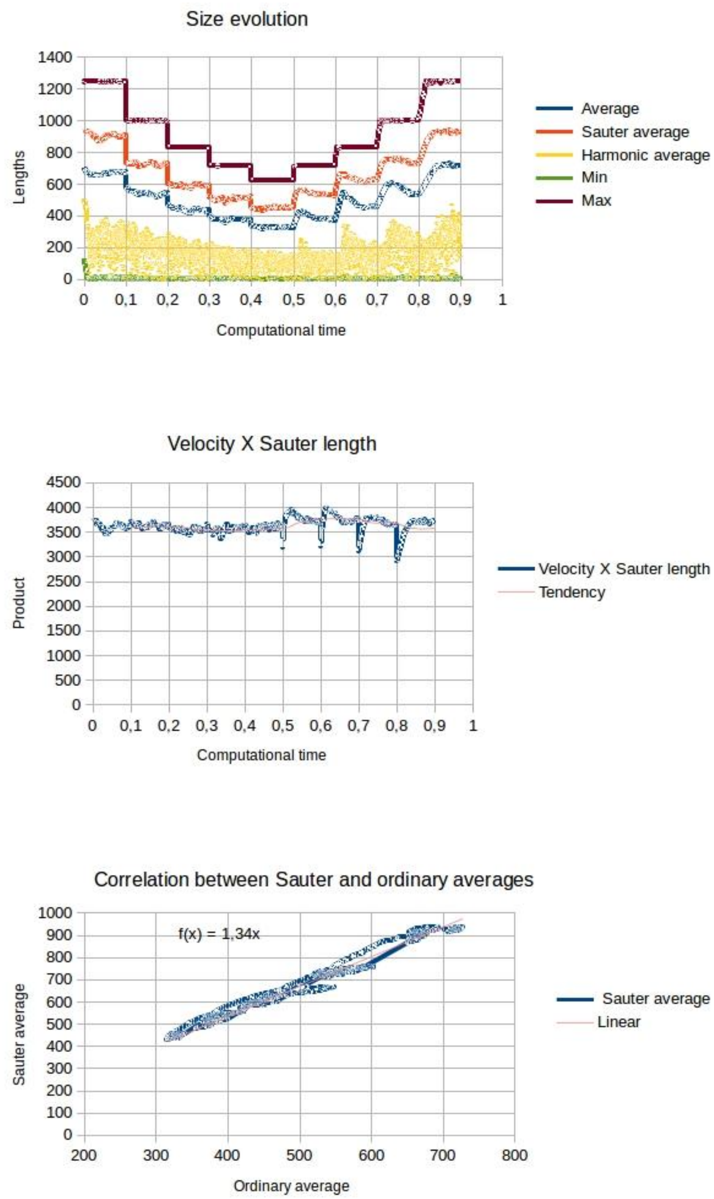

Fig. 9. Evolution of the sizes of scallops when the velocity increases, then decreases
Up: evolution of the mean lengths
Middle: fluctuations of the product used in the Curl relationship
Down: correlations between Sauter and ordinary means

3.3 Evolution with Velocity Variations

This is a very important point: it is widely admitted that, in the case of long-time velocity variations (for instance due to climate change or tectonics), the velocity recorded in the scallops is the last one. However, as large scallops can remain stable if the velocity decreases, the velocity recorded in the scallops may also be, under certain circumstances, the slower one. We had developed such an hypothesis in [11].

Two numerical simulations have been run: one with an initial velocity of *8* cm/s decreasing step by step to *4* cm/s then increasing to *8* cm/s (Fig. 8) and one with an initial velocity of 4 cm/s increasing to *8* mm/s then decreasing to *4* cm/s (Fig. 9).

In both simulations, once a steady state is reached, the size of the scallops reflects the present velocity and not the slower one. This leads us to retract the hypothesis developed in [11]. However, two details must be pointed out. First, when a steady state is not reached yet, the size of the scallops do not correspond, through the Curl relationship, to the present velocity. Second, what happens when the velocity decreases is not symmetrical of what happens when it increases. In the first case, fluctuations are more important and there is a long-time trend to over-estimate the product velocity X Sauter length. In other words, there may be a tendency to misestimate the velocity deduced from the Sauter length: reconstituting past flows using present scallops may remain an imprecise operation.

Both simulations predict a good correlation between the Sauter average and the ordinary average, the later being about *34%* higher than the former. Establishing such a correlation is not our main aim but it could be a very simple way to compare field data and numerical results.

4. CONCLUSION

The dynamics of interaction between neighbouring scallops fully explains the link between flow velocity and statistical properties, without taking in account the precise details of scallops formation and stability. In the case of steady states, through the Curl relationship, there is a perfect match between the Sauter length issued from the modelling and the flow velocity.

This model relies upon a very limited number of parameters. It doesn't need any diffusion coefficient, thermal diffusivity or solubility rate. Only four parameters are needed: the limits of the stability interval, the ratio r describing how a too large scallop splits, the exponent α.

Practically, roughly tuning only two of these parameters (the upper and lower limits of the stability interval) has been enough to produce preliminary realistic results. In addition, running the programs does neither require a long computation time or very powerful cluster-computing.

We can simulate what could happen when velocity variations occur, which is not possible with static models. This has lead us to the same conclusion than in [11]: Certain past velocities may not be precisely accessible knowing only the present Sauter length.

Examining the detailed statistical distribution of scallops, flutes and related two or one-dimensional corrosion forms might lead to further progress.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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