Asian Research Journal of Mathematics

Volume 19, Issue 7, Page 56-64, 2023; Article no.ARJOM.98895
ISSN: 2456-477X

2-Secure Domination in Bipolar Hesitancy Fuzzy Graph

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/ARJOM/2023/v19i7679

Open Peer Review History:
This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/98895

Received: 13/02/2023
Accepted: 15/04/2023
Published: 29/04/2023

Original Research Article

Abstract

We establish secure domination in bipolar hesitancy fuzzy graph. We extend the concept to secure total domination and 2- secure domination in bipolar hesitancy fuzzy graph. Further some theorems and examples related to secure domination are discussed.

Keywords: Bipolar fuzzy graph; hesitant fuzzy graph; domination number; secure domination; 2-Secure domination.

2010 Mathematics Subject Classification: 03E72, 05C72, 05C69.

1 Introduction

L.A. Zadeh [?] was the one who initially introduced the idea of fuzzy sets. By applying Zadeh's fuzzy relation, Kaufmann designed fuzzy graph in 1973. A. Somasundaram and S. Somasundaram were the first to develop

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the fuzzy graph domination notion [2]. Bipolar Fuzzy Graph (BFG) is a concept that was developed by M. Akram. M.G. Karunambigai, Palanivel, and Akram introduced the approach of domination in bipolar fuzzy graphs [3]. The book *"Graphs for the Analysis of Bipolar Fuzzy Information"* by Akram, Sarwar, and Dudek is an excellent resource for comprehending the ideas of domination in BFGs[4].In bipolar fuzzy graphs, S. Ramya and S. Lavanya established edge domination, [5] V. Torra [6] initially developed the idea of hesitant fuzzy sets in the year 2010. Hesitancy fuzzy graph, a new approach to fuzzy graph theory was first established by T. Pathinathan,et.al[7]. R. Sakthivel et al. studied the concept of domination in hesitancy fuzzy graphs[8]. K. Anantha Kanaga Jothi and K. Balasangu defined the concept of irregular and entirely irregular bipolar hesitancy fuzzy graphs in the year 2021, as well as some of its attributes.Also some important paper for reference here [9] [10][11][12][13][14][3][15].

Motivated by these domination concepts, we aim to establish the concept of secure domination in bipolar hesitancy fuzzy graph(BHFG), also discuss some definitions and properties related to 2-secure domination in BHFG with examples.

2 Preliminaries

Definition 2.1. Let $\mathcal{X}$ be a non empty set. A *bipolar fuzzy set* B in $\mathcal{X}$ is an object having the form $B = \{(x, \mu_B^P(x), \mu_B^N(x)) | x \in \mathcal{X}\}$ where, $\mu_B^P : \mathcal{X} \to [0,1]$ and $\mu_B^N : \mathcal{X} \to [-1,0]$ are mapppings.

Definition 2.2. A *Bipolar Fuzzy Graph* (BFG) is of the form $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where

1. $\mathcal{V} = \{v_1, v_2, ...v_n\}$ such that $\mu_1^P : \mathcal{V} \to [0,1]$ and $\mu_1^N : \mathcal{V} \to [-1,0]$
2. $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ where $\mu_2^P : \mathcal{V} \times \mathcal{V} \to [0,1]$ and $\mu_2^N : \mathcal{V} \times \mathcal{V} \to [-1,0]$ such that

$$\mu_2^P(v_i, v_j) \leq min(\mu_1^P(v_i), \mu_1^P(v_j))$$

and

$$\mu_2^N(v_i, v_j) \geq max(\mu_1^N(v_i), \mu_1^N(v_j))$$

for all $(v_i, v_j) \in \mathcal{E}$.

Definition 2.3. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a BFG is said to be *strong* then $\mu_2^P = min(\mu_1^P(v_i), \mu_1^P(v_j))$ and $\mu_2^N = max(\mu_1^N(v_i), \mu_1^N(v_j))$ $\forall v_i, v_j \in \mathcal{V}$.

Definition 2.4. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a BFG is said to be *complete* then,

$$\mu_2^P(v_i, v_j) = min(\mu_1^P(v_i), \mu_1^P(v_j))$$

$$\mu_2^N(v_i, v_j) = max(\mu_1^N(v_i), \mu_1^N(v_j))$$

for all $v_i, v_j \in \mathcal{V}$.

Definition 2.5. An arc (a, b) is said to be strong edge in a BFG, if

$$\mu_2^P(a, b) \geq (\mu_2^P)^\infty(a, b) \text{ and } \mu_2^N(a, b) \geq (\mu_2^N)^\infty(a, b)$$

whereas $(\mu_2^P)^\infty(a, b) = max\{(\mu_2^P)^k(a, b) | k = 1, 2, ..., n\}$ and $(\mu_2^N)^\infty(a, b) = min\{(\mu_2^N)^k(a, b) | k = 1, 2, ..., n\}$.

Definition 2.6. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a BFG, then cardinality of $\mathcal{G}$ is defined as

$$|\mathcal{G}| = \sum_{v_i \in \mathcal{V}} \frac{(1 + \mu_1^P(v_i) + \mu_1^N(v_i))}{2} + \sum_{(v_i, v_j) \in \mathcal{E}} \frac{(1 + \mu_2^P(v_i, v_j) + \mu_2^N(v_i, v_j))}{2}$$

.

Definition 2.7. The cardinality of $\mathcal{V}$, i.e., amount of nodes is termed as the order of $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and is signified by $|\mathcal{V}|$(or $O(\mathcal{G})$) and determined by

$$O(\mathcal{G}) = |\mathcal{V}| = \sum_{v_i \in \mathcal{V}} \frac{(1 + \mu_1^P(v_i) + \mu_1^N(v_i))}{2}$$

The no. of elements in a set of S, i.e., amount of edges is termed as size of $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and signified as $|S|$(or $S(\mathcal{G})$) and determined by

$$S(\mathcal{G}) = |S| = \sum_{(v_i, v_j) \in \mathcal{E}} \frac{(1 + \mu_2^P(v_i, v_j) + \mu_2^N(v_i, v_j))}{2}$$

for all $(v_i, v_j) \in \mathcal{E}$.

3 Bipolar Hesitancy Fuzzy Graph

Definition 3.1. Let $\mathcal{X}$ be a non-empty set. A Bipolar hesitancy fuzzy set

$$\mathtt{B} = \{x, \mu_1^P(x), \mu_1^N(x), \gamma_1^P(x), \gamma_1^N(x), \beta_1^P(x), \beta_1^N(x)/x \in \mathcal{X}\}$$

where $\mu_1^P, \gamma_1^P, \beta_1^P : \mathcal{X} \to [0, 1]$ and $\mu_1^N, \gamma_1^N, \beta_1^N : \mathcal{X} \to [-1, 0]$ are mappings such that,

$$0 \leq \mu_1^P(x) + \gamma_1^P(x) + \beta_1^P(x) \leq 1$$

and

$$-1 \leq \mu_1^N(x) + \gamma_1^N(x) + \beta_1^N(x) \leq 0$$

.

Definition 3.2. Let $\mathcal{X}$ be a non empty set.Then we call mappings $\mu_2^P, \gamma_2^P, \beta_2^P : \mathcal{X} \times \mathcal{X} \to [0, 1]$, $\mu_2^N, \gamma_2^N, \beta_2^N : \mathcal{X} \times \mathcal{X} \to [-1, 0]$ are bipolar hesitancy fuzzy relation on $\mathcal{X}$ such that,
$\mu_2^P(x, y) \leq \mu_1^P(x) \wedge \mu_1^P(y); \mu_2^N(x, y) \geq \mu_1^N(x) \vee \mu_1^N(y);$ $\gamma_2^P(x, y) \leq \gamma_1^P(x) \wedge \gamma_1^P(y); \gamma_2^N(x, y) \geq \gamma_1^N(x) \vee \gamma_1^N(y);$ $\beta_2^P(x, y) \leq \beta_1^P(x) \wedge \beta_1^P(y); \beta_2^N(x, y) \geq \beta_1^N(x) \vee \beta_1^N(y)$.

Definition 3.3. A bipolar hesitancy fuzzy relation A on $\mathcal{X}$ is called symmetric relation if $\mu_2^P(x, y) = \mu_2^P(x, y)$, $\mu_2^N(x, y) = \mu_2^N(x, y)$,$\gamma_2^P(x, y) = \gamma_2^P(x, y)$, $\gamma_2^N(x, y) = \gamma_2^N(x, y)$, $\beta_2^P(x, y) = \beta_2^P(x, y)$, $\beta_2^N(x, y) = \beta_2^N(x, y)$ for all $(x, y) \in \mathcal{X}$.

Definition 3.4. A Hesitancy fuzzy graph is of the form $G = (V, E)$ where,
$V = \{v_!, v_2, \ldots v_n\}$ such that $\mu_1, \gamma_1, \beta_1 : V \to [0, 1]$ denote the degree of membership, non-membership and hesitancy of the vertex $v_i \in V$ respectively and $\mu_1(v_i) + \gamma_1(v_i) + \beta_1(v_i) = 1$ for every $v_i \in V$ where $\beta_1(v_i) = 1 - [\mu_1(v_i) + \gamma_1(v_i)]$ and
$E \subseteq V \times V$ where $\mu_2, \gamma_2, \beta_2 : V \times V \to [0, 1]$ denote the degree of membership, non-membership and hesitancy of the edge $(v_i, v_j) \in E$ respectively such that,
$\mu_2(v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$; $\gamma_2(v_i, v_j) \leq \gamma_1(v_i) \vee \gamma_1(v_j)$; $\beta_2(v_i, v_j) \leq \beta_1(v_i) \wedge \beta_1(v_j)$ and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) + \beta_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$.

Definition 3.5. A Bipolar Hesitancy Fuzzy Graph (BHFG) is of the form $\mathtt{G} = (\mathtt{V}, \mathtt{E})$ where

(i) $\mathtt{V} = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1^P, \gamma_1^P, \beta_1^P : \mathtt{V} \to [0, 1]$ denote the degree of positive membership, positive non-membership and positive hesitancy of the vertex $v_i \in \mathtt{V}$ respectively, $\mu_1^N, \gamma_1^N, \beta_1^N : \mathtt{V} \to [-1, 0]$ denote the degree of negative membership,negative non-membership and negative hesitancy of the vertex $v_i \in \mathtt{V}$.
For every $v_i \in \mathtt{V}$,
$\mu_1^P(v_i) + \gamma_1^P(v_i) + \beta_1^P(v_i) = 1$ and $\mu_1^N(v_i) + \gamma_1^N(v_i) + \beta_1^N(v_i) = -1$
$\beta_1^P(v_i) = 1 - [\mu_1^P(v_i) + \gamma_1^P(v_I)]$ and $\beta_1^N(v_i) = -1 - [\mu_1^N(v_i) + \gamma_1^N(v_i)]$

(ii) $\mathtt{E} \subseteq \mathtt{V} \times \mathtt{V}$ where, $\mu_2^P, \gamma_2^P, \beta_2^P : \mathtt{V} \times \mathtt{V} \rightarrow [0,1]$; $\mu_2^N, \gamma_2^N, \beta_2^N : \mathtt{V} \times \mathtt{V} \rightarrow [-1,0]$ are mappings such that

$$\mu_2^P(v_i, v_j) \leq \mu_1^P(v_i) \wedge \mu_1^P(v_j)$$
$$\mu_2^N(v_i, v_j) \geq \mu_1^N(v_i) \vee \mu_1^N(v_j)$$
$$\gamma_2^P(v_i, v_j) \leq \gamma_1^P(v_i) \vee \gamma_1^P(v_j)$$
$$\gamma_2^N(v_i, v_j) \geq \gamma_1^N(v_i) \wedge \gamma_1^N(v_j)$$
$$\beta_2^P(v_i, v_j) \leq \beta_1^P(v_i) \wedge \beta_1^P(v_j)$$
$$\beta_2^N(v_i, v_j) \geq \beta_1^N(v_i) \vee \beta_1^N(v_j)$$

denote the degree of positive, negative membership, degree of positive, negative non membership and degree of positive, negative hesitancy of the edge $(v_i, v_j) \in \mathtt{E}$ respectively and

$$0 \leq \mu_2^P(v_i, v_j) + \gamma_2^P(v_i, v_j) + \beta_2^P(v_i, v_j) \leq 1$$

,

$$-1 \leq \mu_2^N(v_i, v_j) + \gamma_2^N(v_i, v_j) + \beta_2^N(v_i, v_j) \leq 0$$

for every $(v_i, v_j) \in \mathtt{E}$.

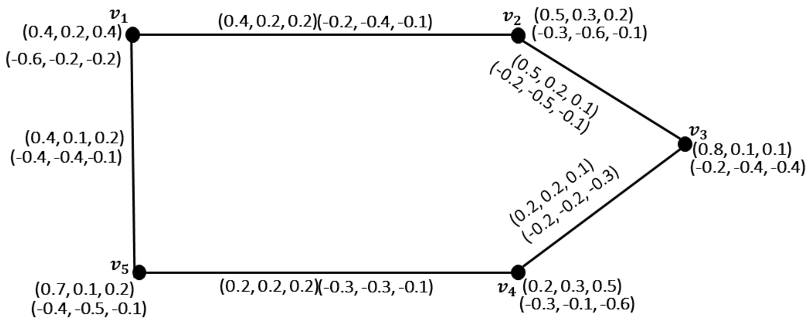

Fig. 1. Bipolar Hesitancy Fuzzy Graph

Example 3.1. *From Fig 2, for vertex* v_1,
$\mu_1^P(v_1) + \gamma_1^P(v_1) + \beta_1^P(v_1) = 0.4 + 0.2 + 0.4 = 1$
$\mu_1^N(v_1) + \gamma_1^N(v_1) + \beta_1^N(v_1) = -0.6 - 0.2 - 0.2 = -1.$
For edge (v_1, v_2); $\mu_2^P(v_1, v_2) + \gamma_2^P(v_1, v_2) + \beta_2^P(v_1, v_2) = 0.8 \leq 1$
$\mu_2^N(v_1, v_2) + \gamma_2^N(v_1, v_2) + \beta_2^N(v_1, v_2) = -0.7 \geq -1.$

Definition 3.6. A Bipolar Hesitancy Fuzzy Graph $\mathtt{G} = (\mathtt{V}, \mathtt{E})$ is said to be complete when, $\mu_2^P(v_i, v_j) = \mu_1^P(v_i) \wedge \mu_1^P(v_j)$,$\mu_2^N(v_i, v_j) = \mu_1^N(v_i) \vee \mu_1^N(v_j)$, $\gamma_2^P(v_i, v_j) = \gamma_1^P(v_i) \vee \gamma_1^P(v_j)$, $\gamma_2^N(v_i, v_j) = \gamma_1^N(v_i) \wedge \gamma_1^N(v_j)$, $\beta_2^P(v_i, v_j) = \beta_1^P(v_i) \wedge \beta_1^P(v_j)$, $\beta_2^N(v_i, v_j) = \beta_1^N(v_i) \vee \beta_1^N(v_j)$ for every $v_i, v_j \in \mathtt{V}$.

Definition 3.7. A Bipolar Hesitancy Fuzzy Graph $\mathtt{G} = (\mathtt{V}, \mathtt{E})$ is said to be strong when, $\mu_2^P(v_i, v_j) = \mu_1^P(v_i) \wedge \mu_1^P(v_j)$,$\mu_2^N(v_i, v_j) = \mu_1^N(v_i) \vee \mu_1^N(v_j)$ $\gamma_2^P(v_i, v_j) = \gamma_1^P(v_i) \vee \gamma_1^P(v_j)$, $\gamma_2^N(v_i, v_j) = \gamma_1^N(v_i) \wedge \gamma_1^N(v_j)$, $\beta_2^P(v_i, v_j) = \beta_1^P(v_i) \wedge \beta_1^P(v_j)$, $\beta_2^N(v_i, v_j) = \beta_1^N(v_i) \vee \beta_1^N(v_j)$ for every $(v_i, v_j) \in \mathtt{E}$.

Definition 3.8. Let $\mathtt{G}$ be a Bipolar hesitancy fuzzy graph. The neighbourhood of a vertex x in $\mathtt{G}$ is defined by $N(x) = (N_\mu^P(x), N_\mu^N(x), N_\gamma^P(x), N_\gamma^N(x), N_\beta^P(x), N_\beta^N(x))$ where
$N_\mu^P(x) = \{y \in \mathtt{V}/\mu_2^P(x,y) \leq \mu_1^P(x) \wedge \mu_1^P(x)\}$; $N_\mu^N(x) = \{y \in \mathtt{V}/\mu_2^N(x,y) \geq \mu_1^N(x) \vee \mu_1^N(x)\}$;$N_\gamma^P(x) = \{y \in \mathtt{V}/\gamma_2^P(x,y) \leq \gamma_1^P(x) \wedge \gamma_1^P(x)\}$; $N_\gamma^N(x) = \{y \in \mathtt{V}/\gamma_2^N(x,y) \geq \gamma_1^N(x) \vee \gamma_1^N(x)\}$;$N_\beta^P(x) = \{y \in \mathtt{V}/\beta_2^P(x,y) \leq \beta_1^P(x) \wedge \beta_1^P(x)\}$; $N_\beta^N(x) = \{y \in \mathtt{V}/\beta_2^N(x,y) \geq \beta_1^N(x) \vee \beta_1^N(x)\}$.

Definition 3.9. Let G be a Bipolar Hesitancy Fuzzy Graph. The neighborhood degree of a vertex x in G is defined by

$$deg(x) = [\deg\mu^P(x), \deg\mu^N(x), \deg\gamma^P(x), \deg\gamma^N(x), \deg\beta^P(x), \deg\beta^N(x)]$$

$y \in \mathtt{V}$, where

$$\deg\mu^P(x) = \sum_{y\in N(x)} \mu_1^P(y), \deg\mu^N(x) = \sum_{y\in N(x)} \mu_1^N(y), \deg\gamma^P(x) = \sum_{y\in N(x)} \gamma_1^P(y)$$

$$\deg\gamma^N(x) = \sum_{y\in N(x)} \gamma_1^N(y), \deg\beta^P(x) = \sum_{y\in N(x)} \beta_1^P(y), \deg\beta^N(x) = \sum_{y\in N(x)} \beta_1^N(y)$$

.

Definition 3.10. Let $\mathtt{G} = (\mathtt{V}, \mathtt{E})$ be a BHFG. The edge cardinality of G is given by,

$$|\mathtt{E}| = r = \sum_{(v_i,v_j)\in \mathtt{E}} \frac{3 + \mu_2^P(v_i,v_j) + \mu_2^N(v_i,v_j) + \gamma_2^P(v_i,v_j) + \gamma_2^N(v_i,v_j) + \beta_2^P(v_i,v_j) + \beta_2^N(v_i,v_j)}{3}$$

.

Definition 3.11. An Arc (u,v) is said to be strong edge in BHFG. then, $\mu_2^P(u,v) \geq (\mu_2^P)^\infty(u,v)$,$\mu_2^N(u,v) \geq (\mu_2^N)^\infty(u,v)$,$\gamma_2^P(u,v) \geq (\gamma_2^P)^\infty(u,v)$,$\gamma_2^N(u,v) \geq (\gamma_2^N)^\infty(u,v)$,$\beta_2^P(u,v) \geq (\beta_2^P)^\infty(u,v)$, $\beta_2^N(u,v) \geq (\beta_2^N)^\infty(u,v)$ whereas $(\mu_2^P)^\infty(u,v) = max\{(\mu_2^P)^k(u,v)|k = 1,2,\ldots,n\}$;$(\mu_2^N)^\infty(u,v) = min\{(\mu_2^N)^k(u,v)|k = 1,2,\ldots,n\}$;$(\gamma_2^P)^\infty(u,v) = max\{(\gamma_2^P)^k(u,v)|k = 1,2,\ldots,n\}$; $(\gamma_2^N)^\infty(u,v) = min\{(\gamma_2^N)^k(u,v)|k = 1,2,\ldots,n\}$; $(\beta_2^P)^\infty(u,v) = max\{(\beta_2^P)^k(u,v)|k = 1,2,\ldots,n\}$; $(\beta_2^N)^\infty(u,v) = min\{(\beta_2^P)^k(u,v)|k = 1,2,\ldots,n\}$.

4 Main Result

Definition 4.1. Let G be a BHFG and $u, v \in \mathtt{V}$. A subset D of V is called dominating set in G if for every $u \in \mathtt{V} - \mathtt{D}$, there exists $u \in \mathtt{D}$ such that u dominates v. The minimum cardinality taken over all dominating sets of G is called the domination number of Gand denoted by $\gamma_b(\mathtt{G})$

Definition 4.2. Let G be a BHFG without isolated vertices. A Total dominating set D of a BHFG G is a dominating set in which the subgraph $< \mathtt{D} >$ induced by D has no isolated vertices. The minimum cardinality taken over all total dominating sets is called the total domination number of G and is denoted as $\gamma_{bt}(\mathtt{G})$.

Definition 4.3. In a BHFG G. A Secure dominating set $\mathtt{S} \subseteq \mathtt{V}$ is a dominating set, if for every vertex $u \in \mathtt{V} - \mathtt{S}$ is adjacent to a vertex $v \in \mathtt{S}$ such that $(\mathtt{S} - \{v\})\, U\{u\}$ is also a dominating set. The minimum cardinality taken over all secure dominating sets of G is called the secure domination number of G and is expressed as $\gamma_{bs}(\mathtt{G})$.

Example 4.1. *From the above graph 4,* $\{v_1, v_2, v_4\}$, ,$\{v_2, v_4, v_5\}$,$\{v_1, v_3, v_4\}$,$\{v_2, v_3, v_4, v_5\}$ *are the secure dominating sets*

Corollary 4.2. *Every secure dominating set will be a dominating set but every dominating set need not be secure.*

Proof. From Example 4.1, The vertex set $\{v_1, v_2\}$ is a dominating set but not a secure dominating set. □

Definition 4.4. Consider a BHFG G without isolated vertices. A *total secure dominating set* is a secure dominating set S in which the subgraph $< \mathtt{S} >$ induced by S has no isolated vertices. The minimum fuzzy cardinality taken over all secure total dominating sets of G is called the total secure domination number of G and is denoted by $\gamma_{bst}(\mathtt{G})$.

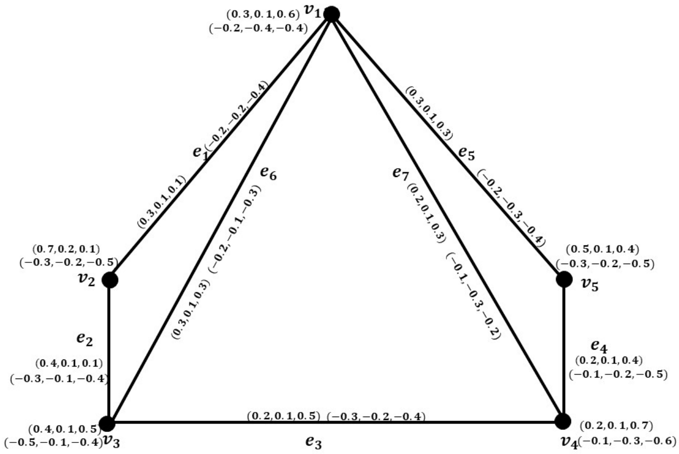

Fig. 2. Secure Domination in BHFG

Definition 4.5. A subset D^* of V is called a *2-dominating set* in G if every vertex of $V - D^*$ has atleast two neighbour in D^*.
The minimum cardinality taken over all 2-dominating sets of G is called the 2-domination number of G and is denoted by $\gamma_2(G)$

Definition 4.6. A subset D^* of V is called a *2-total dominating set* in G, if D^* is a 2-dominating set and the subgraph induced by D^* has no isolated vertices.
The minimum cardinality taken over all 2-total dominating sets of G is called the 2-total domination number of G and is denoted by $\gamma_{2t}(G)$.

Definition 4.7. In a BHFG G. A *secure 2-dominating set* is a 2-dominating set $S^* \subseteq V$, if for every vertex $u \in V - S^*$ is adjacent to a vertex $v \in S^*$ such that $(S^* - \{v\})U\{u\}$ is 2-dominating set.The The minimum cardinality taken over all 2-secure dominating sets of G is called the 2-secure domination number of G and is expressed as $\gamma_{2bs}(G)$.

Definition 4.8. Consider a BHFG G without isolated vertices. A *2-secure total dominating set* is a 2-secure dominating set in which the subgraph $< S^* >$ induced by S^* has no isolated vertices. The minimum fuzzy cardinality taken over all 2-secure total dominating sets of G is called the 2-secure total domination number of G and is denoted by $\gamma_{2bst}(G)$.

Example 4.3. *From the above graph 4, $\{v_1, v_3, v_5, v_6\}$, ,$\{v_2, v_4, v_5, v_6\}$,$\{v_1, v_2, v_3, v_4\}$ are the 2-secure dominating sets*

Theorem 4.4. *Let G be a complete BHFG. If S is a minimal dominating set in G then*

1. *S is a secure dominating set*
2. *S is not a secure total dominating set.*

Proof. Given S is a minimal dominating set of a complete BHFG G. Every arc in a complete bipolar hesitancy fuzzy graph is a strong arc,then minimal dominating set S contains only one vertex v, i.e., $S = \{v\}$. Now for any vertex $v_i \in V - S$ and v_i is adjacent to v. Then $(S - \{v\}) \cup \{v_i\} = \{v_i\}$ is a dominating set. Thus, S is secure dominating set.

Since any secure dominating set of a complete BHFG contains a vertex v_i, by the definition of total dominating, S is not a secure total dominating set. □

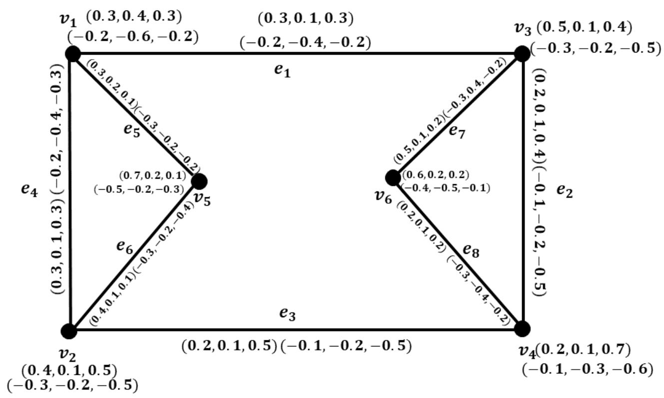

Fig. 3. 2-Secure Domination in BHFG

Theorem 4.5. *Let* G *be a complete BHFG. If* D *is a minimal dominating set in* G *then*

1. D *is not a 2-dominating set,*
2. D *is not a 2-total dominating set.*

Proof. Consider a complete BHFG G, If D is a minimal dominating set in G, then D contains a vertex of minimum cardinality but 2-dominating set should contain atleast two vertices. Therefore, D is not a 2-dominating set. Similarly, D is not a 2-total dominating set. □

Theorem 4.6. *For a complete BHFG* G,

$$\gamma_{bs}(\mathtt{G}) = \gamma_b(\mathtt{G}) \tag{4.1}$$

Proof. Let us consider a complete BHFG G. Let S be a minimal dominating set of G. Then S contains a vertex $\{v\}$, i.e.,

$$\mathtt{S} = \{v\} \tag{4.2}$$

The minimum cardinality of S is denoted by $\gamma_b(\mathtt{G})$. By Theorem (4.5), S is also secure dominating set and the minimum cardinality of secure dominating set is denoted by $\gamma_{bs}(\mathtt{G})$.
Hence, $\gamma_{bs}(\mathtt{G}) = \gamma_b(\mathtt{G})$ □

Theorem 4.7. *Every 2-secure dominating set of a BHFG* G *is a secure dominating set of* G.

Proof. Let G be a BHFG and S be a 2-secure dominating set of G. Then every vertex $u \in \mathtt{V} - \mathtt{S}$ is adjacent to a vertex $v \in \mathtt{S}$ such that $(\mathtt{S} - \{v\}) \cup \{u\}$ is 2-dominating set. Since S is a 2-secure dominating set then by definition, S is a 2-dominating set and every 2-dominating set is a dominating set. Thus every vertex $u \in \mathtt{V} - \mathtt{S}$ is adjacent to a vertex $v \in \mathtt{S}$ such that $(\mathtt{S} - \{v\}) \cup \{u\}$ is a dominating set. Thus S is a secure dominating set of G. □

Theorem 4.8. *For a bipolar hesitancy fuzzy graph* G,

$$\gamma_{2bs}(\mathtt{G}) \geq \gamma_{bs}(\mathtt{G})) \tag{4.3}$$

Proof. By Theorem(4.7), every 2-secure dominating set of a BHFG $\mathtt{G}$ is a secure dominating set of $\mathtt{G}$. Thus every minimum 2-secure dominating set of $\mathtt{G}$ is also a secure dominating set of $\mathtt{G}$. Thus, $\gamma_{2bs}(\mathtt{G}) \geq \gamma_{bs}(\mathtt{G}))$. □

Theorem 4.9. *Let* $\mathtt{G}$ *be a BHFG. If* $\mathtt{S}$ *is a 2-dominating set of a path of* $\mathtt{G}$*, then* $\mathtt{S}$ *is not 2-secure dominating set.*

Proof. Consider a BHFG $\mathtt{G}$.Let P_n be a path of $\mathtt{G}$ and $\mathtt{S}$ is a 2-dominating set of a path P_n. Then $\mathtt{S}$ contain two pendent vertices v_i and v_j. Now for some $u \in \mathtt{V} - \mathtt{S}$ and u is adjacent to v_i. Thus $(\mathtt{S} - \{v_i\}) \cup u$ is not 2-dominating set. Thus $\mathtt{S}$ is not 2-secure dominating set. □

Theorem 4.10. *Let* $\mathtt{G}_{m,n}$ *be a complete bipartite BHFG. If* $\mathtt{S}$ *is a dominating set of* $\mathtt{G}$*, then* $\mathtt{S}$ *is not a secure dominating set.*

Proof. Given that $\mathtt{S}$ is a dominating set of a complete bipartite BHFG $\mathtt{G}_{m,n}$ Then $\mathtt{S}$ should contain a vertex in $\mathtt{V}_1$ say u and a vertex in $\mathtt{V}_2$ say v. Now for some $v_i \in \mathtt{V} - \mathtt{S}$ and v_i is adjacent to $u \in \mathtt{V}_1$. Thus $\{\mathtt{S} - \{u\}\} \cup \{v_i\}$ is not dominating set. So $\mathtt{S}$ is not a secure dominating set. □

Theorem 4.11. *Let* $\mathtt{G}$ *be a BHFG with only strong edges and without isolated vertices and* $\mathtt{S}$ *is a minimal secure dominating set. Then* $\mathtt{V} - \mathtt{S}$ *is a secure dominating set of* $\mathtt{G}$.

Proof. Consider a BHFG $\mathtt{G}$ with only strong edges and without isolated vertices. Given that $\mathtt{S}$ is a minimal secure dominating set of $\mathtt{G}$. Then by definition, every vertex $u \in \mathtt{V} - \mathtt{S}$ is adjacent to a vertex $v \in \mathtt{S}$ such that $(\mathtt{S} - \{v\}) \cup \{u\}$ is dominating set.

Claim: Prove that $\mathtt{V} - \mathtt{S}$ is a secure dominating set of $\mathtt{G}$.

Assume that $\mathtt{V} - \mathtt{S}$ is not secure dominating set. Then there exist vertex $w \in \mathtt{S}$ is adjacent to a vertex $x \in \mathtt{V} - \mathtt{S}$ such that $(\mathtt{S} - \{x\}) \cup \{w\}$ is not dominating set. Thus x is not dominated by any vertex in $\mathtt{S}$ which is contradiction to our assumption that $\mathtt{S}$ is minimal secure dominating set and $\mathtt{G}$ has no isolated vertices and has only strong edges. So $\mathtt{V} - \mathtt{S}$ is a secure dominating set of $\mathtt{G}$. □

5 Applications

As an extension of secure domination in BHFG, the results for 2-secure domination in BHFG were also achieved.These ideas have a wide range of practical applications, including traffic signals where, in the event that any traffic is caused by congestion, a secure line can be constructed for simple vehicle movement by preventing congestion can be made utilising these ideas.

Future developments on this notion include its extension to other concepts of dominations, such as edge and inverse dominations in different types of bipolar fuzzy graphs.

6 Conclusions

Using a bipolar hesitancy fuzzy graph, secure domination was achieved, and in this study, we have outlined the ideas involved. With examples, we have illustrated the ideas.

Acknowledgement

The authors thank the editor and referees for their helpful recommendations that helped to improve the manuscript in its current form.

Competing Interests

Authors have declared that no competing interests exist.

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