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A New Structure and Contribution in D-metric Spaces

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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Abstract

In this paper, we define a new topological structure of D-closed, D-continuous and D-fixed point property and discussed of its properties, some result for this subject are also established.

Keywords: $D-$metric; supra topology.

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1 Introduction

The concept of a D-metric space was introduced by Dhage in [1]. A nonempty set X, together with a function $D : X \times X \times X \to [0, \infty)$ is called a D-metric space, denoted by (X, D) if D satisfies the followings:

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D_1) $D(x, y, z) = 0$ if and only if $x = y = z$ (coincidence),

D_2) $D(x, y, z) = D(p(x, y, z))$, where p is a permutation of x, y, z (symmetry),

D_3) $D(x, y, z) \leq D(x, y, a) + D(x, a, z) + D(a, y, z)$ for all x, y, z,$a \in X$ (tetrahedral inequality).

The nonnegative real function D is called a D-metric on X. Dhage [1] claimed that D-metric convergence defines a Hausdorff topology and that the D-metric is (sequentially) continuous in all the three variables. Many authors (see [2]-[11] and [12]-[14]) have taken these claims for granted and used them in proving fixed point theorems in D-metric spaces. For more discussion, we refer the reader to consrlt ([15]-[21] and [22], [23]). Authors of [24] gave examples to show that in a D-metric space:

1) D-metric convergence does not always define a topology.
2) Even when D-metric convergence defines a topology, it need not be Hausdorff.
3) Even when D-metric convergence defines a metrizable topology, the D-metric need not be continuous even in a single variable.

Definition 1.1. [1], [8]. A sequence $\{x_n\}$ in a D-metric space (X, D) is said to be convergent (or D-convergent) if there exists an element x in X such that $\lim_{n,m} D(x_n, x_m, x) = 0$, i.e. for any $\epsilon > 0$, there exists $j \in \mathbb{N}$ such that $D(x_n, x_m, x) < \epsilon$ for all $n, m \geq j$. In such a case, $\{x_n\}$ is said to converge to x and x is called a limit of $\{x_n\}$. We shall use the notation $\{x_n\} \overset{D}{\to} x$ to denote that $\{x_n\}$ is D-convergent to x.

Definition 1.2. [1], [8]. A sequence $\{x_n\}$ in a D-metric space (X, D) is said to be Cauchy (or D-Cauchy) if, for any $\epsilon > 0$, there exists $j \in \mathbb{N}$ such that $D(x_n, x_m, x_k) < \epsilon$ for all $n, m, k \geq j$.

Definition 1.3. [1], [8]. A D-metric space (X, D) is said to be complete (or D-complete) if every D-Cauchy sequence in X is D-convergent in X.

We shall use the same notation used in [24]. for A^c ,namely.

Notation 1.1. *[24]. For a subset A of a D-metric space (X, D) , A^c denotes the set $\{x \in X :$ there exists $x_n \in A$ such that $\{x_n\} \overset{D}{\to} x\}$. For any set X, $P(X)$ denotes the power set of X.*

S. V. R. Naidu, K.P. R. Rao, and N. Srinivasa Rao have obtained the following nice example.

Example 1.2. *[24]. Let $X = A \cup B \cup \{0\}$, where $A = \{2^{-n} : n \in \mathbb{N}\}$ and $B = \{2^n : n \in \mathbb{N}\}$. Then there exists a D-metric on X such that:*

(i) (X, D) is a complete D-metric space in which D-metric convergence does not define a topology.

(ii) There are convergent sequences in X with infinitely many limits.

(iii) The operator $\varphi : P(X) \to P(X)$ defined by $\varphi(A) = A^c$ does not define a closure operator. More precisely,$(B^c)^c \neq B^c$.

2 D-closed Sets

If A is any subset of a D-metric space (X, D) and if $a \in A$, then $\{x_n\} \overset{D}{\to} a$, where $x_n = a$ for $n \in \mathbb{N}$, because $\lim_{n,m} D(x_n, x_m, a) = 0$, Thus $A \subseteq A^c$. Imitating the case of sequentially closed sets we have the following.

Definition 2.1. A subset E of a D-metric space (X, D) is said to be D-closed provided $E = E^c$ (equivalently $E^c \subseteq E$), i.e. if for any $x_n \in E$ and $p \in X$, if $\{x_n\} \overset{D}{\to} p$, then $p \in E$. The complement of a D-closed set is called D-open. A set in (X, D) will be called D-clopen set if it is D-closed and D-open simultaneously.

The following results are easy to observe.

Proposition 2.1. *Let (X, D) be a D-metric space. If $\{x_n\} \overset{D}{\to} p$ then every D-open set H containing p must contain a tail of $\{x_n\}$.*

Proof. Let H be a D-open set in (X, D) containing p. Suppose on the contrary, that H does not contain any tail of $\{x_n\}$.Then there exists a sequence of natural numbers $1 < m_1 < m_2 < \cdots$ such that $x_{m_n} \notin H$ for all $n \in \mathbb{N}$. Since $\{x_n\} \overset{D}{\to} p$ therefore $\{x_{m_n}\} \overset{D}{\to} p$. Since $X - H$ is D-closed and $x_{m_n} \in X - H$, therefore $p \in X - H$ which is absurd.

Proposition 2.2. *Every finite set in a D-metric space (X, D) must be D-closed.*

Proof. Let A be a finite subset of X and let $x_n \in A$,$p \in X$ such that $\{x_n\} \overset{D}{\to} p$. Then $\lim_{n,m} D(x_n, x_m, x) = 0$, yields the existence of a natural number j such that $x_n = p$ for all $n \geq j$ (i.e. $\{x_n\}$ has a constant tail $p, p, p, \cdots$). Hence $p \in A$.

Proposition 2.3. *Let $D : X \times X \times X \to [0, \infty)$ be a D-metric on X having a finite range. Then every subset A of X is D-closed.*

Proof. Similar to the proof of Proposition 2.2.

Theorem 2.1. *The intersection of any collection of D-closed sets in a D-metric space (X, D) is D-closed.*

Proof. Let $\Im = \{F_\alpha : \alpha \in \Delta\}$ be a collection of D-closed sets in X and let $x_n \in \bigcap_{\alpha\in\Delta} F_\alpha$, $p \in X$ such that $\{x_n\} \overset{D}{\to} p$. Since $x_n \in F_\alpha$, $\{x_n\} \overset{D}{\to} p$ and F_α is a D-closed set, therefore $p \in F_\alpha$ $(\alpha \in \Delta)$. Hence $p \in \bigcap_{\alpha\in\Delta} F_\alpha$. Consequently, $\bigcap_{\alpha\in\Delta} F_\alpha$ is D-closed.

Now it is meaningful to have the following definition.

Definition 2.2. If A is a subset of a D-metric space (X, D) , we define the D-closure of A (denoted by $D - cl(A)$ or $cl_D(A)$) as the intersection of all D-closed sets in X containing A.

It is clear that $D - cl(A)$ is the smallest D-closed set in X containing A. It is also clear that an arbitrary union of D-open sets in a D-metric space (X, D) is still D-open. The fact that the subsets ϕ and X are D-clopen in (X, D) is obvious. Finally, if A is a subset of B then $cl_D(A) \subseteq cl_D(B)$.

Definition 2.3. [24]. A subfamily T^* of X is said to be a supra topology on X if:

(1) ϕ, $X \in T^*$.

(2) If $F_\alpha \in T^*$, $\alpha \in \Delta$, then $\bigcup_{\alpha\in\Delta} F_\alpha \in T^*$

(X, T^*) is called a supra topological space. The elements of T^* are called supra open sets in (X, T^*) and complement of a supra open set is called a supra closed set.

Definition 2.4. [24]. The supra closure of a set A is denoted by supra $cl(A)$ and is defined as supra $cl(A) = \cap\{B : B \textit{ is a supra closed set and } A \subseteq B\}$. The supra interior of a set A is denoted by supra int(A), and defined as supra $int(A) = \cup\{B : B \textit{ is a supra open set and } B \subseteq A\}$.

Theorem 2.2. *Let (X, D) be a D-metric space and $T_D^* = \{A \subseteq X : A \text{ is } D-\text{open set in } (X, D)\}$. Then we have the following.*

(i) T_D^* *is a supra topology on* X.

(ii) Every finite subset of X *is supra closed in* (X, T_D^*).

(iii) For any A *subset of* X, $cl_D(A) = supra\ cl(A)$.

Example 1.2 verifies the following result.

Theorem 2.3. *There exists a D-metric space* (X, D) *such that:*

(i) There exists $B \subseteq X$ *such that* $cl_D(cl_D B) \neq cl_D(B)$.

(ii) There exist subsets M, P *of* X *such that* $cl_D(M \cup P) \neq cl_D(M) \cup cl_D(P)$.

Proof. Indeed , if Proposition 2.3 (ii) fails then (X, T_D^*) will be a topological space and this is absurd. As in usual metric space (X, d) , the collection $\Im(d)$ of all open balls is indeed a subbase for the metric topology $T(d)$. Fortunately, $\Im(d)$is a base for $T(d)$. In a similar way, the collection $\Im^*(D)$ of all D-open sets in a D-metric space (X, D) is a subbase for some topology $T(D)$ on X. Unfortunately, $\Im^*(d)$ need not be a base for some topology on X. So, one starts looking for properties of this topology $T(\Im^*(d))$ generated by $\Im^*(d)$ as a subbase (rather than a base!).

Finally, in this section, we have the following result.

Corollary 2.4. *.Let* $D : X \times X \times X \to [0, \infty)$ *be a D-metric on* X *having a finite range. Then every subset* A *of* X *is D-closed, i. e,* D *generates the discrete topology on* X.

Proof. Immediate consequence of Proposition 2.3.

3 D-continuous Functions

If $(X, \Im)$ and (Y, T) are topological spaces such that $(X, \Im)$ is first countable at $p \in X$. Then a function $f : (X, \Im) \to (Y, T)$ is continuous at p if and only if f is sequentially continuous at p (i.e. for any sequence $\{x_n\}$ converging in $(X, \Im)$ to p, then $\{f(x_n)\}$ must converge in (Y, T) to $f(p)$).

Imitating this idea in D-metric spaces, we get the following.

Definition 3.1. Let $f : (X, D) \to (Y, \rho)$ be a function between two D-metric spaces. Then f is said to be D_ρ-continuous at $p \in X$ provided that for any sequence $\{x_n\}$ converging in $(X, \Im)$ to p, then $\{f(x_n)\}$ must converge in (Y, ρ) to $f(p)$. A function $f : (X, D) \to (Y, \rho)$ is called D_ρ-continuous if f is D_ρ-continuous at each p in X. In the case $X = Y$ and $D = \rho$, we write D-continuous instead of D_ρ-continuous.

Definition 3.2. Let $f : (X, D) \to (Y, \rho)$ be a function between two D-metric spaces. Then f is said to be D_ρ-weakly continuous at $p \in X$ provided that for any ρ-open set H in Y containing $f(p)$, there exists a D-open set U in X containing p such that $f(U) \subseteq H$. A function $f : (X, D) \to (Y, \rho)$ is called D_ρ-weakly continuous if f is D_ρ-weakly continuous at each p in X. In the case $X = Y$ and $D = \rho$, we write D-weakly continuous instead of D_ρ-weakly continuous.

The following result is an analogue to a well-known result in general topology.

Theorem 3.1. *The followings are equivalent for the function* $f : (X, D) \to (Y, \rho)$ *between two D-metric spaces.*

(i) *f is D_ρ-weakly continuous.*

(ii) *For any ρ-open set H in Y, $f^{-1}(H)$ is D-open set in X.*

(iii) *For any ρ-closed set M in Y, $f^{-1}(M)$ is D-closed set in X.*

Proof. (i) $\Rightarrow$ (ii):Let H be a ρ-open set in Y. For each $x \in f^{-1}(H)$, then $f(x) \in H$. Since f is D_ρ-weakly continuous at x, there exists a D-open set U_x such that $x \in U_x$ and $f(U_x) \subseteq H(i.e.U_x \subseteq f^{-1}(H))$. Consequently, $f^{-1}(H)$ is a union of D-open sets in X, and hence is D-open.

(ii) $\Rightarrow$ (iii):Let M be a ρ-closed set in Y. Then $Y - M$ is ρ-open set in Y and hence $f^{-1}(Y - M)$ is a D-open set in X. Consequently, $X - f^{-1}(Y - M)$ is a D-closed set in X, i.e.$f^{-1}(M)$ is a D-closed set in X.

(iii) $\Rightarrow$ (i): To show that f is D_ρ-weakly continuous function ,let $p \in X$ and H be any ρ-open set in Y such that $f(p) \in H$. Then $Y - H$ is a ρ-closed set in Y. Hence $f^{-1}(Y - H)$ is a D-closed set in X. Thus $X - f^{-1}(Y - H)$ is a D-open set in X containing p. It is clear that $f(X - f^{-1}(Y - H)) \subseteq H$. Hence f is D_ρ-weakly continuous.

Theorem 3.2. *Let $f : (X, D) \to (Y, \rho)$ be a function between two D-metric spaces. If f is D_ρ-continuous then f is D_ρ-weakly continuous.*

Proof. Let M be any ρ-closed set in Y. To prove that $f^{-1}(M)$ is D-closed in X, let $\{x_n\}$ be any sequence in $f^{-1}(M)$ converging in X to p. Then the sequence $\{f(x_n)\}$ is in M and converging in Y to $f(p)$.The fact that M is ρ-closed set in Y forces $f(p)$ to belong to M. Hence $p \in f^{-1}(M)$.

4 D-fixed Point Property

We start this section with the following.

Definition 4.1. (i) A D-metric space (X, D) is said to have the D-fixed point property (abbreviated D-f.p.p.) iff every D-continuous function $f : (X, D) \to (X, D)$ has a fixed point in X.

(ii)A D-metric space (X, D) is said to have the D-weakly fixed point property (abbreviated D-$w.f$.p.p.) iff every D-weakly continuous function $f : (X, D) \to (X, D)$ has a fixed point in X.

The following result is an immediate consequence of Theorem 3.2.

Corollary 4.1. *If a D-metric space has the D-$w.f$.p.p. then it has the D-f.p.p.*

Definition 4.2. A D-metric space (X, D) is called D-disconnected provided there exists a partition $\{A, B\}$ for X consisting of two D-closed sets in X. (X, D) is called D-connected provided it is not D-disconnected.

Definition 4.3. Let (X, D) be a D-metric space and A, B be two nonempty subsets of X. Then A, B are called D-separated sets provided $A \cap cl_D(B) = B \cap cl_D(A) = \emptyset$.

The following result is needed for the next theorem.

Lemma 4.2. *In a D-metric space (X, D), if A, B are two D-separated sets in X such that $A \cup B = X$. Then each of them is a D-clopen set in X.*

Proof. Since $B \subseteq cl_D(B)$ and $A \cap cl_D(B) = \emptyset$,therefore $A \cap B = \emptyset$. Now $A \cup B = X$ and $A \cap B = \emptyset$ implies $A = X - B$. The fact that $cl_D(B) \subseteq X - A = B$ implies $B = cl_D(B)$,i.e. B is D-closed. Similarly, A is a D-closed set in X. Consequently, A and B are D-clopen sets in X.

Now the proof of the following result becomes easy to follow.

Theorem 4.3. *The following conditions are equivalent for a D-metric space (X, D).*

(i) (X, D) *is D-disconnected.*

(ii) There exists a D-clopen set A in X such that $\emptyset \neq A \neq X$.

(iii) X has a partition consisting of two D-open sets.

(iv) X has a nontrivial D-separation.

(v) There exists a surjective D_ρ-continuous function $f : (X, D) \to (\{0, 1\}, \rho)$, where ρ is any D-metric on $\{0, 1\}$.

The following result will be needed later.

Theorem 4.4. *Let (X, D) be a D-metric space and $\{A, B\}$ be a partition of X consisting of D-closed (D-open) sets in X. Let $a \in A$ and $b \in B$ be fixed elements. Then the function $f : (X, D) \to (X, D)$ defined by: $f(A) = \{b\}$ and $f(B) = \{a\}$, is D-continuous(and hence D-weakly continuous).*

Proof. To prove that f is D-continuous, let $\{x_n\}$ be any sequence in X and assume $\{x_n\} \overset{D}{\to} p$. Without loss of generality we may assume $p \in A$. To prove $\{f(x_n)\} \overset{D}{\to} f(p)$ we are going to show that $\lim_{n,m} D(f(x_n), f(x_m), f(p)) = 0$. To prove this, we claim that there exists $j \in \mathbb{N}$ such that for all $n \geq j$, $f(x_n) = b$ (i.e. $x_n \in A$ for n sufficiently large). To prove our claim, suppose not , i.e. for each $j \in \mathbb{N}$ there exists $n_j \in \mathbb{N}$ such that $n_j > n$ and $x_{n_j} \notin A$ (hence $x_{n_j} \in B$). Doing this, we can find an infinite sequence of natural numbers $1 < n_1 < n_2 < \cdots$ such that $x_{n_j} \in B$ for all $j \in \mathbb{N}$. Since $\lim_{n,m} D(x_n, x_m, p) = 0$ therefore $\lim_{i,j} D(x_{n_i}, x_{n_j}, p) = 0$. Thus $\{x_{n_j}\} \overset{D}{\to} p$. Now $x_{n_j} \in B$ and B is D-closed in X implies that $p \in B$, this is a contradiction. Now since our claim becomes valid ,i.e. a tail of $\{x_n\}$ must be in A. Hence $f(x_n) = b$ for all n large enough. Thus $\lim_{n,m} D(f(x_n), f(x_m), f(p)) = 0$, i.e. $\{f(x_n)\} \overset{D}{\to} f(p)$.

Definition 4.4. A D-metric space (X, D) is called a D-T_0-space provided that for any x, $y \in X$, $x \neq y$,there exists a D-open set H such that $H \cap \{x, y\}$ has exactly one element.

The following results are now ready to be proved.

Theorem 4.5. *(i) If (X, D) has the D-$f.p.p.$, then it is D-connected.*
(ii) If (X, D) has the D-$w.f.p.p.$, then it is D-connected.

Proof. (i) Let (X, D) be a D- metric space having the D-fixed point property. Suppose on the contrary, that X is D-disconnected. Then X has a partition $\{A, B\}$ consisting of D-closed sets in X. Now pick $a \in A$ and $b \in B$. Define $f : (X, D) \to (X, D)$ by: $f(A) = \{b\}$ and $f(B) = \{a\}$.Then f is D-continuous according to Theorem 4.4.Notice that f has no fixed point and this contradicts the assumption that (X, D) has the D-$w.f$.p.p.
(ii) The proof is a direct consequence of (i) together with Proposition 4.2.

Theorem 4.6. *(i) If (X, D) has the D-$f.p.p.$, then it is a D-T_0-space.*
(ii) If (X, D) has the D-$w.f.p.p.$, then it is a D-T_0-space.

Proof. (i) Let (X, D) be a D- metric space having the D-fixed point property. Suppose on the contrary, that X is not a D-T_0-space. Then there exist p, $q \in X$, $p \neq q$ such that for any D-open set H in X, either $H \cap \{p, q\} = \emptyset$ or $\{p, q\} \subseteq H$. Define $f : (X, D) \to (X, D)$ by:

$$f(x) = \begin{cases} p & \text{if } x \neq p \\ q & \text{if } x = p \end{cases}$$

Then f clearly has no fixed point. To prove f is D-continuous, let H be any D-open set in X. Then $f^{-1}(H) = \begin{cases} X & \text{if } p \in H \\ \emptyset & \text{if } p \in X - H \end{cases}$. Notice that for any D-open set H in X we have: $q \in H$ if and only if $p \in H$. Hence f is D-continuous, which gives us the contradiction.
(ii)The proof is a direct consequence of (i) together with Proposition 4.2.

5 Product of D-metric Spaces

Let us start with the following.

Proposition 5.1. *Let (X_i, D_i) be D-metric spaces$(i = 1, \cdots, n)$ and let $X = X_1 \times \cdots \times X_n$. Define $D : X \times X \times X \to [0, \infty)$ as follows:*

$$D((x_1, \cdots, x_n), (y_1, \cdots, y_n), (z_1, \cdots, z_n)) = \sum_{i=1}^{n} D_i(x_i, y_i, z_i).$$

Then D is a D- metric on X (we shall denote D by $\sum_{i=1}^{n} D_i$).

Proof. Straight forward.

Proposition 5.2. *Let (X, D) be a D- metric space and $\emptyset \neq A \subseteq X$. Define $D^* : A \times A \times A \to [0, \infty)$ as follows: $D^*(x, y, z) = D(x, y, z)$. Then D^* is a D-metric on A (D^* will be denoted by $D^{(A)}$).*

Proof. Straight forward.

The following result will be needed later.

Lemma 5.1. *Let (X_i, D_i) be D-metric spaces$(i = 1, \cdots, n)$ and let $X = X_1 \times \cdots \times X_n$. Define $D : X \times X \times X \to [0, \infty)$ as in Proposition 5.1. Let $z_k = (x_k^{(1)}, x_k^{(2)}, \cdots, x_k^{(n)}) \in X \quad (k \in \mathbb{N})$. Then $\{z_k\} \stackrel{D}{\to} z = (p_1, p_2, \cdots, p_n)$ if and only if $\{x_k^j\} \stackrel{D_j}{\to} p_j \ (j = 1, 2, \cdots, n)$.*

Proof. Straight forward.

Theorem 5.2. *Let (X_i, D_i) be D-metric spaces$(i = 1, \cdots, n)$ and let $X = X_1 \times \cdots \times X_n$. Define $D : X \times X \times X \to [0, \infty)$ as in Proposition 5.1. If A_i is a D_i-closed set in (X_i, D_i) for $i = 1, \cdots, n$. Then $A_1 \times A_2 \times \cdots \times A_n$ is a D-closed set in (X, D).*

Proof. The proof is an immediate consequence of Lemma 5.1.

6 Conclusion

In this paper, we have given the notion of a new topological structure of D-closed, D-continuous and D-xed point property and discussed of its properties, some result for this subject are also established. We hope that our results can also be extended to other topological field.

Competing Interests

Authors have declared that no competing interests exist.

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