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Combined Solitary Wave Solutions in Higher-order Effects Optical Fibers

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Abstract

We construct in this manuscript, the combined solitary wave solutions of nonlinear Schrödinger equation that governs the dynamics of propagation of waves in optical fibers with higher-order effects. We base our survey on the sum of two analytic shapes of the solitary waves of bright and dark type to form a resulting solitary wave to determine.

Keywords: Combined solitary wave; BDKm; higher order effects optical fibers; nonlinear Schrödinger equations.

1 Introduction

The physical systems are generally governed by the partial differential equations that are in most cases nonlinear. When these partial differential equations are linear, they obey the principles of superposition and especially of the uniqueness of the solution. When these partial differential equations are nonlinear, the principle of linear combination of the solutions is not applicable and the approach in the resolution varies according to the type of equation and especially of the type of solutions that one wants to obtain. Thus, it becomes difficult to speak of standard method of resolution or unique solution. It is exactly for this reason that the multiple methods and different approaches developed by many authors exist [1-16] with the aim of solving the considered equation completely or to find the approached solutions with a minimal error margin.

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Beyond all these methods of integration of the nonlinear partial differential equations, the research of the forced solutions occupies an important place. To adapt and to exploit the natural phenomena which surround us in order to improve our daily work. Among the searched solutions, the solitary wave probably occupies a very important place because of its futuristic applications. This fact is translated in many articles written by many researchers [17-22]. In this dynamics, we construct some solitary wave solutions of the nonlinear partial differential equation that govern the propagation of waves in the higher nonlinear effects optical fibers and govern by the equation [23,24].

$$U_z - i\alpha_1 U_{tt} - i\alpha_2 |U|^2 U - \alpha_3 U_{ttt} - \alpha_4 \left(|U|^2 U\right)_t - \alpha_5 U\left(|U|^2\right)_t = 0, \tag{1}$$

where U is the slowly varying envelope of the electric field, the subscripts z and t denote the spatial and temporal partial derivatives, and α_1, α_2, α_3, α_4 and α_5 are real constants related to the group-velocity dispersion, the self-phase modulation, the third-order dispersion, the self-steepening, and the delayed nonlinear response effect, respectively. The solutions that we construct effectively are of type combined solitary wave; that means resulting from a combination of the bright solitary wave and dark solitary wave. Otherwise to say the solutions that we want to construct are of the shapes

$$U_{lj}(z,t) = \sum_{l,j} a_{lj} \frac{\sinh^j(\alpha t)}{\cosh^l(\alpha t)} \exp\left[-i(kz - \omega t)\right], \tag{2}$$

where a_{lj}, α are constants, k the spatial frequency of the wave and ω the angular frequency of the wave to be determined as a function of the parameters $\alpha_\chi\left(\chi = 1,2,...,5\right)$ of eq.(1), $l = 1,2,...$; $j = 0,1$ and $i^2 = -1$. If we define for example from eq. (2), two solutions as

$$U_1(z,t) = \left(\frac{a}{\cosh(\alpha t)} + b\frac{\sinh(\alpha t)}{\cosh(\alpha t)}\right)\exp\left[-i(kz - \omega t)\right], \tag{3}$$

and

$$U_2(z,t) = \left(\frac{c}{\cosh^2(\alpha t)} + d\frac{\sinh(\alpha t)}{\cosh^2(\alpha t)}\right)\exp\left[-i(kz - \omega t)\right], \tag{4}$$

where the constants a, b, c and d are to be determined, we see that U_1 is constituted by $U_{11} = \left(a/\cosh(\alpha t)\right)\exp\left[-i(kz - \omega t)\right]$ which is a bright solitary wave and $U_{12} = \left(b\sinh(\alpha t)/\cosh(\alpha t)\right)\exp\left[-i(kz - \omega t)\right]$ which is a dark solitary wave. On the other hand U_2 is constituted by $U_{21} = \left(c/\cosh^2(\alpha t)\right)\exp\left[-i(kz - \omega t)\right]$ which is a bright solitary wave and $U_{22} = \left(d\sinh(\alpha t)/\cosh^2(\alpha t)\right)\exp\left[-i(kz - \omega t)\right]$ which is a dark solitary wave.

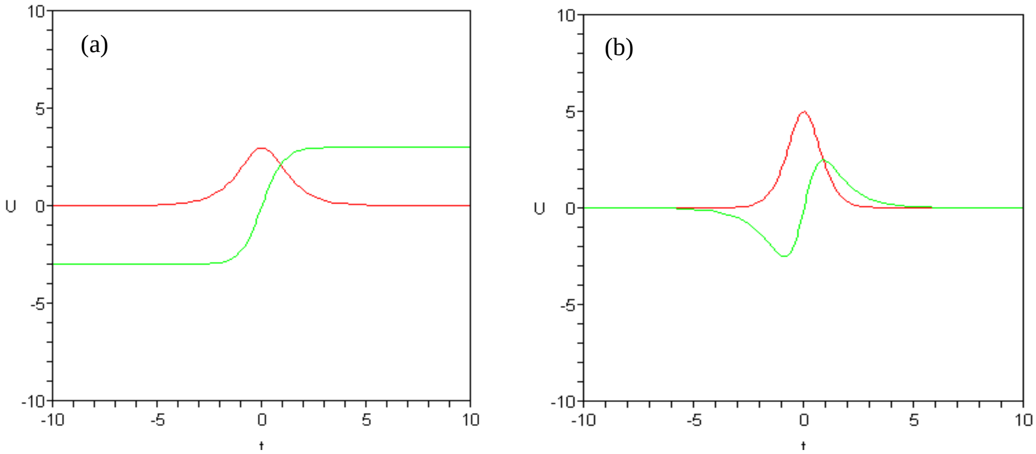

Fig. 1. Curves of $|U_{11}|$, $|U_{12}|$, $|U_{21}|$ and $|U_{22}|$ as a function of time; (a): The red curve indicates the profile of $|U_{11}|$ for $a=5$ and $\alpha=1$; the green curve the profile of $|U_{12}|$ for $b=5$ and $\alpha=1$; (b): The red curve indicates the profile of $|U_{21}|$ for $c=5$ and $\alpha=1$; the green curve gives the profile of $|U_{22}|$ for $d=5$ and $\alpha=1$

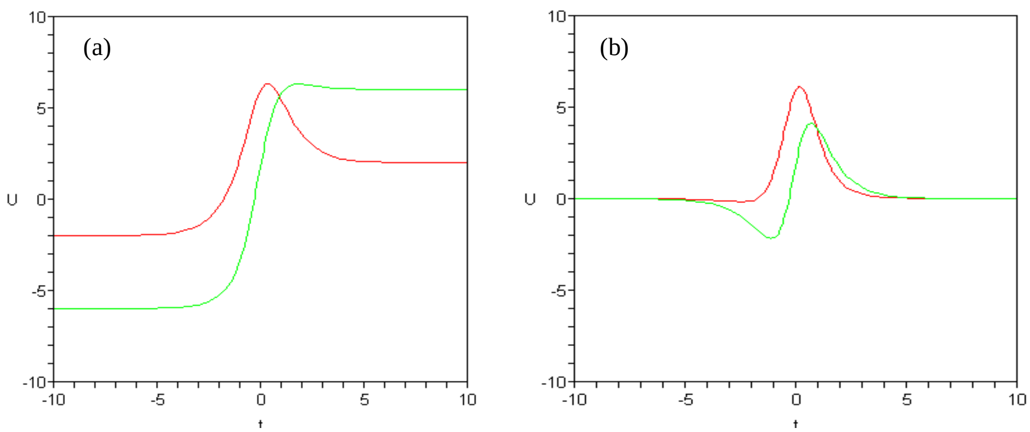

Fig. 2. Curves of $|U_1|$ and $|U_2|$ as a function of time; (a): The red curve indicates the profile of $|U_1|$ for $a=6$, $b=2$ and $\alpha=1$; the green curve the profile of $|U_1|$ for $a=2$, $b=6$ and $\alpha=1$; (b): The red curve indicates the profile of $|U_2|$ for $c=6$, $d=2$ and $\alpha=1$; the green curve gives the profile of $|U_1|$ for $c=2$, $d=6$ and $\alpha=1$

The curves of the Fig. 1 show the profiles of single solitary waves and the curves of the Fig. 2 show the profiles of the combined solitary waves. In the two cases the red curves indicate the bright solitary waves and the green curves indicate the dark solitary waves.

Thus, the combined solutions U_1 and U_2 can be bright or dark type according to the choice of the coefficients a, b, c and d. When $a \succ b$, U_1 is a bright soliton. It becomes a dark soliton when $a \prec b$. For U_2, we have the same findings depending on $c \succ d$ or $c \prec d$. The above diagrams illustrate these different analyses. So in a general manner when we have a resultant solitary wave formed of an association of two solitary waves of type pulse and kink, the profile of the solitary wave obtained is either the one of a pulse or a dark. All depends on the choice of the values of parameters. In some cases the obtained profiles can stretch toward a bright or toward a dark without take the complete shape of the bright or the dark. In the setting of this survey, what interests us is the case where the resultant solitary wave is formed of an association of a pulse and a kink and why? Because the sum of the solitary waves of the same nature gives a resultant solitary wave of this considered nature. These types of solutions as we want to look for can have numerous applications in physics. It is besides this principle of analysis that motivates the construction of the solutions as we propose in this work. We organize the manuscript in the following manner:

Before constructing some solutions under the shape proposed in eq. (2), we look for in section 2 of the possible solutions of the shape $\left(a\sinh^m(\alpha t)/\cosh^n(\alpha t)\right)\exp\left[-i\left(kz-\omega t\right)\right]$. In section 3, we construct the solutions of the type given by eq. (3). Section 4, proposes the solutions of the type given by eq. (4). Finally, section 5 concludes the work.

2 Method of Resolution

The principle consist of searching globally for the solutions of eq. (1) in the form

$$v = \sum_{i,j} a_{ij}\left(\sinh(\alpha x)\right)^j / \left(\cosh(\alpha x)\right)^i, \tag{5}$$

where $i = 0,1,...;$ $j = 0,1,...;$ α and a_{ij} are the coefficients to determine. When we introduce the ansatz (5) in eq. (1) we obtain with the help of adequate transformations [7,8], an equation of the form

$$\begin{aligned}&\sum_{i,j,n} F\left(a_{ij}\right)/\cosh^n(\alpha x) + \sum_{i,j,m} G\left(a_{ij}\right)\sinh(\alpha x)/\cosh^m(\alpha x)\\&+\sum_{i,j,k} H\left(a_{ij}\right)\cosh^k(\alpha x) + \sum_{i,j,l} T\left(a_{ij}\right)\cosh^l(\alpha x)\sinh(\alpha x) + \sum_{i,j} W\left(a_{ij}\right) = 0\end{aligned}. \tag{6}$$

$F\left(a_{ij}\right)$, $G\left(a_{ij}\right)$, $H\left(a_{ij}\right)$, $T\left(a_{ij}\right)$ and $W\left(a_{ij}\right)$ are linear functions of the coefficients a_{ij}. From eq. (6), we obtain the series of equations of constants a_{ij} to solve. Notably the equations such as follows [7,8]:

Term in $1/\cosh^n(\alpha x)$,

$$\sum_{i,j} F\left(a_{ij}\right) = 0, \tag{7}$$

Term in $\sinh(\alpha x)/\cosh^m(\alpha x)$,

$$\sum_{i,j} G\left(a_{ij}\right) = 0, \tag{8}$$

Term in $\cosh^k(\alpha x)$,

$$\sum_{i,j} H(a_{ij}) = 0, \tag{9}$$

Term in $\cosh^l(\alpha x)\sinh(\alpha x)$,

$$\sum_{i,j} T(a_{ij}) = 0, \tag{10}$$

Term in $(\sinh(\alpha x))^0 / (\cosh(\alpha x))^0$,

$$\sum_{i,j} W(a_{ij}) = 0. \tag{11}$$

Finally, the resolution of the above eqs. (7), …, (11) permits to obtain the constants a_{ij}.

3 Results and Discussion

3.1 Pulse or Kink Solitary Wave Solution

In this section we look for the single solitary waves that are the solutions of the eq.(1). Thus, we suppose that

$$U(z,t) = a\frac{\sinh^m(\alpha t)}{\cosh^n(\alpha t)}\exp\left[-i(kz - \omega t)\right], \tag{12}$$

is the solution of eq. (1) where a, α and k are constants to be determined, n and m are the whole numbers to be determined and $i^2 = -1$. The ansatz (12) can represent the analytic shape of the solitary wave of type pulse or kink according to the choice of the parameters n and m. For example $m = 0$ and $n \neq 0$, we have a solitary wave of type pulse. For $m = 1$ and $n \neq 0$, we have a solitary wave of type kink. Then inserting eq. (12) in eq. (1) yields

$$\sum_{n,m} F_{nm}(a,k,\alpha,n,m)\frac{\sinh^m(\alpha t)}{\cosh^n(\alpha t)} = 0, \tag{13}$$

where $F_{nm}(a,k,\alpha,n,m)$ are the function of the constants a, k, α, n and m. The non trivial solutions are only gotten for $m = 0$ and $m = 1$. Thus, the coefficients of the terms in $1/\cosh^3(\alpha t)$, $1/\cosh(\alpha t)$, $\sinh(\alpha t)/\cosh^4(\alpha t)$ and $\sinh(\alpha t)/\cosh^2(\alpha t)$ give respectively

$$\alpha^2(2\omega\alpha_3 - 2\alpha_1) - (\alpha_2 + \omega\alpha_4)|a|^2 = 0, \tag{14}$$

$$i\left[-k+\alpha_1\left(1+\omega^2\right)\alpha^2-2\alpha^2\omega\alpha_3\right]-\alpha_3\omega^2=0, \tag{15}$$

$$6\alpha_3\alpha+\left(3\alpha_4+2\alpha_5\right)|a|^2=0, \tag{16}$$

and

$$2\alpha_1\omega+\left(2\alpha\omega^2+1+\omega^2\right)\alpha^2\alpha_3=0 \tag{17}$$

The resolution of eq. (14) and eq. (16) gives

$$\alpha=\frac{-6\left(\alpha_3\alpha_2+\omega\alpha_3\alpha_4\right)}{6\omega\alpha_3\alpha_4-4\omega\alpha_3\alpha_5-6\alpha_1\alpha_4-4\alpha_1\alpha_5}, \tag{18}$$

and

$$|a|^2=\frac{\alpha^2\left(2\omega\alpha_3-2\alpha_1\right)}{\alpha_2+\omega\alpha_4}, \tag{19}$$

with $6\omega\alpha_3\alpha_4-4\omega\alpha_3\alpha_5-6\alpha_1\alpha_4-4\alpha_1\alpha_5\neq 0$ and $\alpha_2+\omega\alpha_4\neq 0$. On the other hand eq. (15) gives

$$k=\alpha_1\left(1+\omega^2\right)\alpha^2-2\alpha^2\omega\alpha_3+i\alpha_3\omega^2, \tag{20}$$

where α is given by eq. (18). Similarly eq. (17) is written

$$\left(2\alpha^3+\alpha^2\alpha_3\right)\omega^2-2\alpha_1\omega+\alpha^2\alpha_3=0. \tag{21}$$

In the case where the condition $\alpha_1^2-\alpha^2\alpha_3\left(2\alpha^3+\alpha^2\alpha_3\right)\geq 0$ is verified, we obtain from eq. (21)

$$\omega=\frac{\alpha_1\pm\sqrt{\alpha_1^2-\alpha^2\alpha_3\left(2\alpha^3+\alpha^2\alpha_3\right)}}{2\alpha^3+\alpha^2\alpha_3}. \tag{22}$$

Taking into account the eqs. (18), (19), (20) and (22) in eq. (12), we obtain the solution

$$U(z,t)=\pm\alpha\sqrt{\frac{2\omega\alpha_3-2\alpha_1}{\alpha_2+\omega\alpha_4}}\sec h(\alpha t)\exp\left(\alpha_3\omega^2 z\right)\exp\left[-i\left(\Omega z-\omega t\right)\right], \tag{23}$$

with $\Omega=\alpha_1\left(1+\omega^2\right)\alpha^2-2\alpha^2\omega\alpha_3$ such that α and ω are given respectively by eq. (18) and eq. (22).

3.2 Solitary Wave which Results from the Combination of the Solitary Waves of First Order

The substitution of eq. (3) in eq. (1) leads to

$$\sum_{q,p}\left(H_{qp}\left(a,b\right)+iG_{qp}\left(a,b\right)\right)\frac{\sinh^{p}\left(\alpha t\right)}{\cosh^{q}\left(\alpha t\right)}=0,\ q=1,2,...,5;\quad p=0,1 \tag{24}$$

where $H_{qp}\left(a,b\right)$ and $G_{qp}\left(a,b\right)$ are the functions of a and b, $i^2=-1$. From eq.(24), we obtain the series of equations of constants a and b

$$\sum_{q,p}H_{qp}\left(a,b\right)=0, \tag{25}$$

and

$$\sum_{q,p}G_{qp}\left(a,b\right)=0. \tag{26}$$

Some particular solutions of a and b can be obtain by summing eq. (25) and eq. (26). Then eq. (25) and eq. (26) lead to

$$\sum_{q,p}\left(H_{qp}\left(a,b\right)+G_{qp}\left(a,b\right)\right)=0. \tag{27}$$

Thus, from eq. (27), we have

Term in $1/\cosh^4\left(\alpha t\right)$,

$$a^2\alpha\left(\alpha_5+9\alpha_4\right)-\alpha\left(\alpha_5+3\alpha_4\right)b^2+3\alpha_2 ab=0, \tag{28}$$

Term in $1/\cosh^3\left(\alpha t\right)$,

$$b^2-a^2=-\frac{\alpha^2}{\alpha_2}\left(\alpha_3\omega+2\alpha_1\right), \tag{29}$$

Term in $1/\cosh^2\left(\alpha t\right)$,

$$\alpha_4\alpha\left(2b^2-6a^2\right)-\left(3\alpha_2\omega^2+2\alpha_1\omega\right)\alpha-2\alpha_2 a=0, \tag{30}$$

Term in $1/\cosh\left(\alpha t\right)$,

$$\alpha_5\alpha\left(4a^2b-2b^3\right)-\alpha_2 ab^2+\left(a\omega^2-3\alpha_3\omega\alpha^2+\alpha_1\omega^2-\alpha_1\alpha^2-k\right)a=0. \tag{31}$$

Another equations which derive from the term in $\sinh\alpha t/\cosh^l\alpha t$, $l=1,2,...,5$ can allow to establish the constraint relations between the parameters of the physics system governs by eq. (1). Combining eq. (29) and eq. (30), we obtain

$$4\alpha_2\alpha_4\alpha a^2+2\alpha_2^2 a+2\alpha_4\alpha^3+\alpha_2\left(3\alpha_2\omega^2+2\alpha_1\omega\right)\alpha=0. \quad (32)$$

Equation (32) is a quadratic equation in a. It resolution gives

$$a=\frac{-\alpha_2^2\pm\sqrt{\Delta'}}{4\alpha_2\alpha_4}, \quad (33)$$

with $\Delta'=\alpha_2^4-4\alpha_2\alpha_4\alpha^2\left(2\alpha_4\alpha^2+3\alpha_2^2\omega^2+2\alpha_1\alpha_2\omega\right)$ such that $\Delta'\geq 0$. Inserting eq. (33) in eq. (29), we obtain

$$b=\pm\sqrt{\frac{1}{4\alpha_2\alpha_4}\left[\left(-\alpha_2^2\pm\sqrt{\Delta'}\right)^2-16\alpha_2\alpha_4^2\left(\alpha_3\omega+2\alpha_1\right)\right]}. \quad (34)$$

Taking into account eq. (33) and eq. (34) into eq. (3), we obtain

$$U_1(z,t)=\left[\begin{array}{l}\left(\dfrac{-\alpha_2^2\pm\sqrt{\Delta'}}{4\alpha_2\alpha_4}\right)\sec h(\alpha t)\\ \pm\sqrt{\dfrac{1}{4\alpha_2\alpha_4}\left[\left(-\alpha_2^2\pm\sqrt{\Delta'}\right)^2-16\alpha_2\alpha_4^2\left(\alpha_3\omega+2\alpha_1\right)\right]}\tanh(\alpha t)\end{array}\right]\exp\left[-i\left(kz-\omega t\right)\right]. \quad (35)$$

3.3 Solitary Solution Wave which Results from the Combination of the Solitary Waves of Second Order

Substituting ansatz (4) in eq. (1), we obtain the range equation

$$\sum_{\mu,\gamma}R_{\mu\gamma}\left(a,b,\alpha,\omega,k\right)\frac{\sinh^\gamma(\alpha t)}{\cosh^\mu(\alpha t)}+i\sum_{\mu,\gamma}T_{\mu\gamma}\left(a,b,\alpha,\omega,k\right)\frac{\sinh^\gamma(\alpha t)}{\cosh^\mu(\alpha t)}=0, \quad (36)$$

where $R_{\mu\gamma}\left(a,b,\alpha,\omega,k\right)$ and $T_{\mu\gamma}\left(a,b,\alpha,\omega,k\right)$ are functions of constants to determine; $i^2=-1$.

The real part and the imaginary part of eq. (36) lead to the set of equations according to the Bogning-Djeumen Tchaho-Kofané method (BDKm) [7,8]

$$\sum_{\mu,\gamma}R_{\mu,\gamma}\left(a,b,\alpha,\omega,k\right)=0, \quad (37)$$

and

$$\sum_{\mu,\gamma}T_{\mu\gamma}\left(a,b,\alpha,\omega,k\right)=0. \quad (38)$$

The combination of eq. (37) and eq. (38) which derive from the same terms in $1/\cosh^{\mu}(\alpha t)$, $\mu = 1,...,7$ leads to the following equations:

Term in $1/\cosh^{7}(\alpha t)$,

$$4\alpha\left(c^2 d - d^3\right)\left(\alpha_4 + \alpha_5\right) = 0, \tag{39}$$

Term in $1/\cosh^{6}(\alpha t)$,

$$8c^2 d\alpha\left(\alpha_4 + \alpha_5\right) + c\alpha_2\left(c^2 - 3d^2\right) = 0, \tag{40}$$

Term in $1/\cosh^{5}(\alpha t)$,

$$2\left(\alpha_4 + \alpha\alpha_5\right)c^2 - \left(2\alpha_4 + \alpha\alpha_4 + 3\alpha\alpha_5\right)d^2 + 12\alpha_1\alpha^2 = 0, \tag{41}$$

Term in $1/\cosh^{4}(\alpha t)$,

$$2\alpha^2 c^2 d\left(\alpha_4 + \alpha_5\right) - 6\alpha_1\alpha^2 c + 3\alpha_2 d^2 c + 6\alpha_3\alpha^2\omega c = 0, \tag{42}$$

Term in $1/\cosh^{3}(\alpha t)$,

$$4\alpha_1\alpha\omega d - \alpha_3\left(20\alpha^3 d - 6\alpha\omega^2 d\right) + 2\alpha\left(\alpha_4 + \alpha_5\right)d^3 = 0, \tag{43}$$

Term in $1/\cosh(\alpha t)$,

$$\alpha_1\omega\alpha d - \alpha_3\left(-\alpha^3 + 2\alpha\omega^2\right)d = 0. \tag{44}$$

The resolution of eq. (39) supposes three possibilities. The case where $d\left(c^2 - d^2\right) = 0$ and $\alpha_4 + \alpha_5 = 0$, the case where $d\left(c^2 - d^2\right) = 0$ and $\alpha_4 + \alpha_5 \neq 0$ and the case where $d\left(c^2 - d^2\right) \neq 0$ and $\alpha_4 + \alpha_5 = 0$. It is important to mention that we are looking for the non trivial solutions of eq. (1); so a and b must be different from zero and only the case $d \neq 0$, $c \neq d$ and $\alpha_4 + \alpha_5 = 0$ permits to obtain the non trivial solutions. Thus, taking into account eq. (39) into eq. (40) yields

$$c^2 = 3d^2. \tag{45}$$

Introducing eq. (45) in eq. (41) gives

$$d^2 = \frac{12\alpha_1\alpha^2}{\alpha\alpha_4 - 4\alpha_4 - 3\alpha\alpha_5}, \tag{46}$$

and

$$c^2 = \frac{36\alpha_1\alpha^2}{\alpha\alpha_4 - 4\alpha_4 - 3\alpha\alpha_5}, \tag{47}$$

with $\alpha\alpha_4 - 4\alpha_4 - 3\alpha\alpha_5 \neq 0$. Taking into account eq. (46) and eq. (47) into eq. (48) and eq. (49) allows to obtain α as a function of $\alpha_i\left(i = 1, 2, ..., 5\right)$ and consequently the relation which can help to determine ω as a function of a_i such as

$$\begin{aligned} &20\alpha_3\left(24\alpha_3\alpha_4\omega - 24\alpha_1\alpha_4 - 108\alpha_1\right)^2 \\ &= \left(4\alpha_1\omega + 6\alpha_3\omega^2\right)\left(6\alpha_1\alpha_4 - 18\alpha_1\alpha_5 - 6\alpha_3\alpha_4\omega + 18\alpha_3\alpha_4\omega\right)^2. \end{aligned} \tag{48}$$

Introducing the obtained expressions of α and ω in equations eq. (37) and eq. (38), we obtain the expression of k as a function of α_i. Then taking into account eq. (39) and eq. (40) in eq. (4), we obtain

$$U_2\left(z,t\right) = \begin{bmatrix} \pm 6\alpha\sqrt{\dfrac{\alpha_1}{\alpha\alpha_4 - 4\alpha_4 - 3\alpha\alpha_5}}\sec h^2\left(\alpha t\right) \\ \pm\alpha\sqrt{\dfrac{12\alpha_1}{\alpha\alpha_4 - 4\alpha_4 - 3\alpha\alpha_5}}\sinh\alpha t\sec h^2\left(\alpha t\right) \end{bmatrix}\exp\left[-i\left(kz - \omega t\right)\right]. \tag{49}$$

4 Conclusion

The aim of this work was to construct some forced solitary wave solutions. For this reason we carried our choice on the combined solitary wave solutions that means a solitary wave that result from the association of a solitary wave of type pulse and kink. That is a solitary wave which can take according to the choice of the parameters, a shape pulse or kink. In the setting of BDKm used in this work, when one has several equations of the constants, one limits itself to the resolution of the equations of the first range susceptible to produce the solutions that come closer to best of the exact solution. In case of the solution given by the eq. (46), we concentrated our analysis to the range of equations in $1/\cosh^{\mu}\alpha t$ with $\mu = 1, 2, ..., 7$. What spur our interest in the construction of such solutions is that these solutions could have important applications in physics or in engineering of telecommunication. The survey made in this manuscript can be spread to the cases where the resultant solitary wave solution is an association of three solitary waves, four and more.

Competing Interests

Authors have declared that no competing interests exist.

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