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A Block Hybrid Implicit Algorithms for Solution of First Order Differential Equations

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Abstract

The paper consider the derivation of block hybrid algorithms , with k=4 for solution of first order ordinary differential equations, we adopted the method of interpolation and collocation of power series approximation to generate the continuous formula, which was evaluated at off grid and some grid points within the step length to generate the proposed block schemes. Also the schemes obtained were investigated and found to be consistent and zero stable. Finally the method is tested with numerical experiments to ascertain their level of accuracy.

Keywords: Block method; off- grid; higher order; consistent and zero stability.

1 Introduction

Many life and physical problems can be modeled into differential equation of the form

$$y' = f(x, y) \tag{1.0}$$

The equation (1.0) occurs in field of sciences and engineering when we come across physical and natural phenomena which, when represented by mathematical models, happen to be differential equations. Some of these differential equations do not possess the closed form solution hence the numerical computation which is the area of mathematics and computer science that creates analyses and implements algorithms for numerical or approximate solutions is adopted to obtain the solution of (1.0). Many Researchers have worked extensively in this area such as [1,2,3,4,5,6,7], to mention but a few.

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The aim of this research paper is to develop a higher order, zero stable and consistent block method at value k=4, and use it to solve some existing known problems to ascertain the level of their convergence.

Definition 1.0: One-Step Method (see [8]).

The method of constructing of an approximate solution using only one previous value is called one step method. The approach in this method enjoys the virtue that the step size (h) can be changed at every iteration, if desired, thus providing a mechanism for error control. A general expression of one-step method is

$$\sum_{j=0}^{1} \alpha_j y_{n+j} = h \sum_{j=0}^{1} \beta_j f_{n+j} \tag{1.1}$$

Definition 1.1: Linear Multi-step Methods

The general form of linear k −step method for first order ordinary differential equation is

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h \sum_{j=0}^{k} \beta_j f_{n+j} \tag{1.2}$$

where y_n is the numerical approximation to the exact solution at the point x_n and α_1 , α_2,..., α_k , β_1 , β_2 , ..., β_k are fixed numbers .The values of α_1 , α_2 ,... , α_k , β_1 , β_2 , ..., β_k are chosen to obtain the highest possible order of the method.

Definition 1.3: Zero Stability

The linear multistep method (1.2) is said to satisfy the root conditions if all the roots of the first characteristics polynomial have modulus less than or equal to unity and those of modulus unity are simple. The method (1.2) is said to be zero stable if it satisfies the root condition (see [6]).

Theorem 1.0: Fundamental theorem of Dahlquist

The necessary and sufficient conditions for a Linear Multistep Method (LMM) to be convergent are that it must be Consistent and Zero stable.

Theorem 1.1: Dalhquist order barrier for LMM

a. A zero-stable, k-step LMM is maximum order P with $p = (k + 1)$ when k is odd and $k + 2$,when k is even
b. An explicit LMM cannot attain A-stability if the step number, k is such that $k > 2$
c. The Order P of an A-stable LMM cannot exceed two. In fact, the Trapezoidal rule which is of Order p =2 with step number, $k = 1$ known for its A- stability has the smallest Error constant of $C = \frac{1}{12}$ (see [6]).

2 Methodology

We assumed a power series of the form

$$P(x) = \sum_{j=0}^{\infty} \alpha_j x^j$$

which is used as our basis to produce an approximate solution to (1.0) as

$$y(x) = \sum_{j=0}^{m+t-1} \alpha_j x^j \tag{2.1}$$

and

$$y'(x) = \sum_{j=0}^{m+t-1} j\alpha_j x^j = f(x,y) \tag{2.2}$$

where α_j are the parameters to be determined, and $m\ and\ t$ are the points of collocation and interpolation respectively. This process leads to $(m+t-1)$ of non-linear system of equations with $(m+t-1)$ unknown coefficients, which are to be determined by the use of Maple 17 Mathematical software.

2.1 Hybrid block methods derived at $k = 4$

Using equations (2.1) and (2.2), $m = 2, t = 7$ our choice of degree of polynomial is $(m+t-1)$. Equations (2.1) is interpolated at the points $x = \left(x_{n+\frac{3}{2}}, x_{n+\frac{4}{3}}\right)$ and equation (2.2) is collocated at $x = \left(0, \frac{1}{2}, 1, 2, \frac{5}{2}, 3, 4\right)$ which gives the following non-linear system of equations of the form

$$\sum_{j=0}^{s+t-1} \alpha_j x_{n+i}^j = y_{n+i} \qquad \mathrm{i} = \left(\frac{3}{2}, \frac{4}{3}\right) \tag{2.3}$$

$$\sum_{j=1}^{s+t-1} j\alpha_j x_{n+i}^j = f_{n+i} \qquad \mathrm{i} = \left(0, \frac{1}{2}, 1, 2, \frac{5}{2}, 3, 4\right) \tag{2.4}$$

With the mathematical software, we obtain the continuous formulation of equations (2.3) and (2.4) of the form

$$y(x) = \alpha_{\frac{3}{2}} y_{n+\frac{3}{2}} + \alpha_{\frac{4}{2}} y_{n+\frac{4}{2}} + h\left[\beta_0 f_n + \beta_{\frac{1}{2}} f_{n+\frac{1}{2}} + \beta_1 f_{n+1} + \beta_2 f_{n+2} + \beta_{\frac{5}{2}} f_{n+\frac{5}{2}} + \beta_3 f_{n+3} + \beta_4 f_{n+4}\right] \tag{2.5}$$

After obtaining the values of α_j and β_i , j = $\left(\frac{3}{2}, \frac{4}{3}\right)$, i = $\left(0, \frac{1}{2}, 1, 2, \frac{5}{2}, 3, 4\right)$ in (2.5)

and we evaluated it at $x = x_{n+j}\ j = \left(0, \frac{1}{2}, 1, 2, \frac{5}{2}, 3, 4\right)$ and its first derivative also evaluated at $x = x_{n+j}\ j = \frac{3}{2}$, which gives the following set of discrete schemes to form our block hybrid implicit method.

$$y_n = \frac{61210624}{21635123} y_{n+\frac{3}{2}} + \frac{82845747}{21635123} y_{n+\frac{4}{3}} - \frac{15939762}{108175615} hf_n - \frac{579846144}{757229305} hf_{n+\frac{1}{2}} - \frac{1905728}{21635123} hf_{n+1}$$

$$- \frac{10313728}{108175615} hf_{n+\frac{5}{2}} + \frac{4656120}{21635123} hf_{n+2} + \frac{2169792}{108175615} hf_{n+3} - \frac{81758}{151445861} hf_{n+4}$$

$$y_{n+\frac{1}{2}} = \frac{849875}{21635123} y_{n+\frac{3}{2}} + \frac{20785248}{21635123} y_{n+\frac{4}{3}} + \frac{4782875}{1038485904} hf_n - \frac{334507225}{1817350332} hf_{n+\frac{1}{2}} - \frac{2962612775}{4673186568} hf_{n+1}$$

$$+\frac{44498675}{2336593284}hf_{n+\frac{5}{2}}-\frac{5404525}{129810738}hf_{n+2}-\frac{2065585}{519242952}hf_{n+3}+\frac{6852275}{65424611952}hf_{n+4}$$

$$y_{n+1}=-\frac{22299520}{21635123}y_{n+\frac{3}{2}}+\frac{43934643}{21635123}y_{n+\frac{4}{3}}-\frac{14584801}{15577288560}hf_n+\frac{15681458}{1363012749}hf_{n+\frac{1}{2}}-\frac{4599823229}{23365932840}hf_{n+1}$$

$$-\frac{7040042}{584148321}hf_{n+\frac{5}{2}}+\frac{134830361}{3894322140}hf_{n+2}+\frac{17326847}{7788644280}hf_{n+3}-\frac{17027791}{327123059760}hf_{n+4}$$

$$y_{n+2}=\frac{59721728}{21635123}y_{n+\frac{3}{2}}-\frac{38086605}{21635123}y_{n+\frac{4}{3}}-\frac{921571}{649053690}hf_n+\frac{6477344}{454337583}hf_{n+\frac{1}{2}}-\frac{234825218}{2920741605}hf_{n+1}$$

$$-\frac{130058336}{2920741605}hf_{n+\frac{5}{2}}+\frac{101249168}{324526845}hf_{n+2}+\frac{2219618}{324526845}hf_{n+3}-\frac{324526845}{40890382470}hf_{n+4}$$

$$y_{n+\frac{5}{2}}=\frac{47319251}{21635123}y_{n+\frac{3}{2}}-\frac{25684128}{21635123}y_{n+\frac{4}{3}}-\frac{4297153}{15577288560}hf_n+\frac{14969563}{3894322140}hf_{n+\frac{1}{2}}-\frac{154194523}{4673186568}hf_{n+1}$$

$$+\frac{2184623651}{11682966420}hf_{n+\frac{5}{2}}+\frac{506645545}{778864428}hf_{n+2}-\frac{46505851}{7788644280}hf_{n+3}+\frac{517867}{9346373136}hf_{n+4}$$

$$y_{n+3}=\frac{67280000}{21635123}y_{n+\frac{3}{2}}-\frac{45644877}{21635123}y_{n+\frac{4}{3}}-\frac{940125}{346161968}hf_n+\frac{3772770}{151445861}hf_{n+\frac{1}{2}}-\frac{20883875}{173080984}hf_{n+1}$$

$$+\frac{13849250}{21635123}hf_{n+\frac{5}{2}}+\frac{37619625}{86540492}hf_{n+2}+\frac{29887635}{173080984}hf_{n+3}-\frac{1638625}{2423133776}hf_{n+4}$$

$$y_{n+4}=-\frac{634519552}{21635123}y_{n+\frac{3}{2}}+\frac{656154675}{21635123}y_{n+\frac{4}{3}}+\frac{20257960}{194716107}hf_n-\frac{1160673280}{1363012749}hf_{n+\frac{1}{2}}+\frac{1906794880}{584148321}hf_{n+1}$$

$$-\frac{2493900800}{584148321}hf_{n+\frac{5}{2}}+\frac{1225675360}{194716107}hf_{n+2}+\frac{539196800}{194716107}hf_{n+3}+\frac{995496640}{4089038247}hf_{n+4}$$

$$-790091366400y_{n+\frac{3}{2}}+790091366400y_{n+\frac{4}{3}}=266203350hf_n+23223646880hf_{n+\frac{1}{2}}-13917147720hf_{n+1}$$

$$-791479584hf_{n+5/2}+17913605hf_{n+2}+1467011072hf_{n+3}-141948042003hf_{n+4} \qquad (2.6)$$

Equations (2.6) are of uniform order 8, with error constant as follows

$$\left[\frac{1369559}{81780764940},-\frac{148732265}{50246101979136},\frac{948754903}{807526638950400},-\frac{52744349}{323010655580160},\frac{100242059}{39254767171200}\right.$$

$$\left.\frac{58380475}{8374350329856}.-\frac{88610933}{22080865338},-\frac{247098623}{648}\right]^T$$

3 Block Analysis of the Methods

The method in (2.5) is arranged in matrix form as:

$$
\begin{bmatrix}
0 & 0 & -\frac{82845747}{21635123} & -\frac{61210624}{21635123} & 0 & 0 & 0 & 0 \\
1 & 0 & -\frac{20785248}{21635123} & -\frac{849875}{21635123} & 0 & 0 & 0 & 0 \\
0 & 1 & -\frac{43934643}{21635123} & \frac{22299520}{21635123} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{38086605}{21635123} & -\frac{59721728}{21635123} & 1 & 0 & 0 & 0 \\
0 & 0 & \frac{25684128}{21635123} & -\frac{47319251}{21635123} & 0 & 1 & 0 & 0 \\
0 & 0 & \frac{45644877}{21635123} & -\frac{67280000}{21635123} & 0 & 0 & 1 & 0 \\
0 & 0 & -\frac{656154675}{21635123} & \frac{634519552}{21635123} & 0 & 0 & 0 & 1 \\
0 & 0 & 790091366400 & -790091366400 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} y_{n+\frac{1}{2}} \\ y_{n+1} \\ y_{n+\frac{4}{3}} \\ y_{n+\frac{3}{2}} \\ y_{n+2} \\ y_{n+\frac{5}{2}} \\ y_{n+3} \\ y_{n+4} \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} y_{n-\frac{7}{2}} \\ y_{n-3} \\ y_{n-\frac{8}{3}} \\ y_{n-\frac{5}{2}} \\ y_{n-2} \\ y_{n-\frac{3}{2}} \\ y_{n-1} \\ y_{n} \end{bmatrix}
$$

$$
+\begin{bmatrix}
-\frac{579846144}{757229305} & -\frac{1905728}{21635123} & 0 & 0 & \frac{4656120}{21635123} & -\frac{10313728}{108175615} & \frac{2169792}{108175615} & -\frac{81758}{151445861} \\
-\frac{334507225}{1817350332} & -\frac{2962612775}{4673186568} & 0 & 0 & -\frac{5404525}{129810738} & \frac{44498675}{2336593284} & -\frac{2065585}{519242952} & \frac{6852275}{65424611952} \\
\frac{15681458}{1363012749} & -\frac{4599823229}{23365932840} & 0 & 0 & \frac{134830361}{3894322140} & -\frac{7040042}{584148321} & \frac{17326847}{7788644280} & -\frac{17027791}{327123059760} \\
\frac{6477344}{454337583} & -\frac{234825218}{2920741605} & 0 & 0 & \frac{101249168}{324526845} & -\frac{130058336}{2920741605} & \frac{2219618}{324526845} & -\frac{324526845}{40890382470} \\
\frac{14969563}{3894322140} & -\frac{154194523}{4673186568} & 0 & 0 & \frac{506645545}{778864428} & \frac{2184623651}{11682966420} & -\frac{46505851}{7788644280} & \frac{517867}{9346373136} \\
\frac{3772770}{151445861} & -\frac{20883875}{173080984} & 0 & 0 & \frac{37619625}{86540492} & \frac{13849250}{21635123} & \frac{29887635}{173080984} & -\frac{1638625}{2423133776} \\
-\frac{1160673280}{1363012749} & \frac{1906794880}{584148321} & 0 & 0 & \frac{1225675360}{194716107} & -\frac{2493900800}{584148321} & \frac{539196800}{194716107} & \frac{995496640}{4089038247} \\
23223646880 & -13917147720 & 0 & 0 & 17913605 & -791479584 & 1467011072 & -141948042003
\end{bmatrix}
$$

$$
\begin{bmatrix} f_{n+\frac{1}{2}} \\ f_{n+1} \\ f_{n+\frac{4}{3}} \\ f_{n+\frac{3}{2}} \\ f_{n+2} \\ f_{n+\frac{5}{2}} \\ f_{n+3} \\ f_{n+4} \end{bmatrix}
+
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{15939762}{108175615} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4782875}{1038485904} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{14584801}{15577288560} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{921571}{649053690} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{4297153}{15577288560} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{940125}{346161968} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{20257960}{194716107} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 266203350
\end{bmatrix}
\begin{bmatrix} f_{n-\frac{7}{2}} \\ f_{n-3} \\ f_{n-\frac{8}{3}} \\ f_{n-\frac{5}{2}} \\ f_{n-2} \\ f_{n-\frac{3}{2}} \\ f_{n-1} \\ f_{n} \end{bmatrix}
\qquad (3.1)
$$

Let

$$A^{0}=\begin{bmatrix} 0 & 0 & -\frac{82845747}{21635123} & -\frac{61210624}{21635123} & 0 & 0 & 0 & 0 \\ 1 & 0 & -\frac{20785248}{21635123} & -\frac{849875}{21635123} & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{43934643}{21635123} & \frac{22299520}{21635123} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{38086605}{21635123} & -\frac{59721728}{21635123} & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{25684128}{21635123} & -\frac{47319251}{21635123} & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{45644877}{21635123} & -\frac{67280000}{21635123} & 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{656154675}{21635123} & \frac{634519552}{21635123} & 0 & 0 & 0 & 1 \\ 0 & 0 & 790091366400 & -790091366400 & 0 & 0 & 0 & 0 \end{bmatrix}, A^{(1)}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B^{(0)}=\begin{bmatrix} -\frac{579846144}{757229305} & -\frac{1905728}{21635123} & 0 & 0 & \frac{4656120}{21635123} & -\frac{10313728}{108175615} & \frac{2169792}{108175615} & -\frac{81758}{151445861} \\ -\frac{334507225}{1817350332} & -\frac{2962612775}{4673186568} & 0 & 0 & -\frac{5404525}{129810738} & \frac{44498675}{2336593284} & -\frac{2065585}{519242952} & \frac{6852275}{65424611952} \\ \frac{15681458}{1363012749} & -\frac{4599823229}{23365932840} & 0 & 0 & \frac{134830361}{3894322140} & -\frac{7040042}{584148321} & \frac{17326847}{7788644280} & -\frac{17027791}{327123059760} \\ \frac{6477344}{454337583} & -\frac{234825218}{2920741605} & 0 & 0 & \frac{101249168}{324526845} & -\frac{130058336}{2920741605} & \frac{2219618}{324526845} & -\frac{324526845}{40890382470} \\ \frac{14969563}{3894322140} & -\frac{154194523}{4673186568} & 0 & 0 & \frac{506645545}{778864428} & \frac{2184623651}{11682966420} & -\frac{46505851}{7788644280} & \frac{517867}{9346373136} \\ \frac{3772770}{151445861} & -\frac{20883875}{173080984} & 0 & 0 & \frac{37619625}{86540492} & \frac{13849250}{21635123} & \frac{29887635}{173080984} & -\frac{1638625}{2423133776} \\ -\frac{1160673280}{1363012749} & \frac{1906794880}{584148321} & 0 & 0 & \frac{1225675360}{194716107} & -\frac{2493900800}{584148321} & \frac{539196800}{194716107} & \frac{995496640}{4089038247} \\ 23223646880 & -13917147720 & 0 & 0 & 17913605 & -791479584 & 1467011072 & -141948042003 \end{bmatrix}$$

$$B^{(0)}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{15939762}{108175615} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{4782875}{1038485904} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{14584801}{15577288560} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{921571}{649053690} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{4297153}{15577288560} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{940125}{346161968} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{20257960}{194716107} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 266203350 \end{bmatrix}$$

We shall normalize the block method (3.1) by multiplying matrices $A^{(0)}, A^{(1)}, B^{(0)}, B^{(1)}$ with inverse of $A^{(0)}$ to obtain $A'^{(0)}.A'^{(1)}.B'^{(0)}$ *and* $B'^{(1)}$ respectively. By testing the condition of zero stability of $\rho(R) = det\left|RA'^{(0)} - A'^{(1)}\right| = 0$, we have

$$\rho(R) = det\left[R\begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} - \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}\right] = 0$$

$$= det\left(\begin{vmatrix} R & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & R & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & R & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & R & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & R & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & R & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & R & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & R-1 \end{vmatrix}\right) = R^7(R-1) = 0$$

Which implies that $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = 0$ and $R_8 = 1$. Hence from the definition (1.3), the method (3.1) is zero stable and also consistent as its order is $[8,8,8,8,8,8,8,8]^T > 1$ and thus convergent.

4 Numerical Experiments

The block method derived at k = 4 are demonstrated with the following problems:

Problem 1:

$$y' = -y \qquad y(0) = 1, \quad 0 \leq x \leq 1, \quad h = 0.1$$

Exact Solution: $y(x) = e^{-x}$

Problem 2:

$$y' = xy, \qquad y(0) = 1, \quad h = 0.1$$

Exact Solution: $y(x) = e^{\frac{x^2}{2}}$

Problem 3:

$$y' + 4y = 20, \quad y(0) = 2, \quad h = 0.01$$

Exact Solution: $y(x) = 5 - 3e^{-4x}$

Table 1. Comparison of approximate solution of problem 1

x	Exact solution	Computed result	Error in [9]	New error
0.1000	0.9048374180359595	0.904837418035945	7.36E-10	1.4988E-14
0.2000	0.818730753077982	0.818730753077965	4.78E-10	1.70974E-14
0.3000	0.740818220681718	0.740818220681713	4.82E-10	4.996E-15
0.4000	0.670320046035639	0.670320046035336	4.36E-10	3.0298E-13
0.5000	0.606530659712633	0.606530659712355	9.13E-10	2.78E-13
0.6000	0.548811636094026	0.54881163609393	6.94E-10	9.59233E-14
0.7000	0.49658530379141	0.496585303791375	6.91E-10	3.50275E-14
0.8000	0.449328964117222	0.44932896411716	6.17E-10	6.2006E-14
0.9000	0.406569659740599	0.40656965974057	9.41E-10	2.89768E-14
1.0000	0.367879441171442	0.367879441171513	7.71-E10	7.100E-14

5 Discussion of Result

We observed that from all the three problems tested with this proposed block hybrid implicit method there results converges to exact solutions and also compared favourably with the existing similar methods (see Tables 1, 2 and 3).

Table 2. Comparison of approximate solution of problem 2

x	Exact solution	Computed result	Error in [3]	New error
0.1000	1.0050125208594	1.00501252086179	3.7950(-11)	2.38987E-12
0.2000	1.02020134002675	1.0202013400286	3.8550(-11)	1.85008E-12
0.3000	1.04602785990871	1.04602785990969	1.1000(-13)	9.79883E-13
0.4000	1.08328706767495	1.08328706776356	9.4456(-10)	8.861E-11
0.5000	1.13314845306682	1.13314845322051	1.4793(-9)	1.5369E-10
0.6000	1.19721736312181	1.19721736337855	1.3264(-8)	2.5674E-10
0.7000	1.27762131320488	1.27762131362009	5.3520(-8)	4.1521E-10
0.8000	1.37712776433595	1.37712776498828	2.7533(-7)	6.5233E-10
0.9000	1.49930250005676	1.49930250107794	1.3014(-6)	1.02118E-09
1.0000	1.64872127070012	1.648721272298	6.3015(-6)	1.59788E-09

Table 3. Comparison of approximate solution of problem 3

x	Exact solution	Computed result	Error in [7]	New error
0.01	2.11763168254303	2.11763168254301	1.1000(-14)	1.9984E-14
0.02	2.23065096084009	2.23065096084004	-	4.9738E-14
0.03	2.33923868984853	2.33923868984853	2.1000(-14)	-
0.04	2.44356863310137	2.44356863310126	-	1.10134E-13
0.05	2.54380774076605	2.54380774076596	5.1000(-13)	9.01501E-14
0.06	2.64011641680034	2.64011641680036	4.0400(-12)	1.9984E-14
0.07	2.73264877563282	2.73264877563282	3.3010(-11)	-
0.08	2.82155288877893	2.82155288877925	5.6076(-10)	3.19744E-13
0.09	2.90697102178691	2.90697102178704	7.4662(-9)	1.30118E-13
1.00	2.98903986189308	2.98903986189323	8.4977(-8)	1.50102E-13

6 Conclusion

We conclude that our proposed block hybrid implicit method is of uniform order 8 at k = 4. Also the new block method displays its superiority over [3,7,9] from tables of results.

Competing Interests

Authors have declared that no competing interests exist.

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