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Riemannian Curvature Tensor in the Cartesian Coordinate Using the Golden Metric Tensor

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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ABSTRACT

The golden metric tensor completes Euclidean geometry. Since geometry is the foundation of theoretical physics, it implies that our discovery of the golden metric paves way for redefining almost everything in theoretical physics. In this paper, we show how to express the Riemannian curvature tensor in terms of the golden metric tensor for all gravitational fields in nature in the cartesian coordinate. These results which are mathematically most elegant, physically most natural and satisfactory are further used to derive the Riemannian curvature scalar and ricci curvature tensor in the cartesian coordinate.

Keywords: Riemannian; cartesian coordinate; tensor; golden metric.

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1. INTRODUCTION

In the theory of tensor analysis, the Riemannian curvature tensor $R^{\delta}_{\alpha\beta\varphi}$ is defined as [1]:

$$R^{\delta}_{\alpha\beta\varphi} = \Gamma^{\delta}_{\alpha\varphi,\beta} - \Gamma^{\delta}_{\alpha\beta,\varphi} + \Gamma^{\epsilon}_{\alpha\varphi}\Gamma^{\delta}_{\epsilon\beta} - \Gamma^{\epsilon}_{\alpha\beta}\Gamma^{\delta}_{\epsilon\varphi} \tag{1}$$

$$R^{0}_{000} = 0 \tag{2}$$

$$R^{1}_{001} = \Gamma^{1}_{01,0} - \Gamma^{1}_{00,1} + \Gamma^{0}_{01}\Gamma^{1}_{00} + \Gamma^{1}_{01}\Gamma^{1}_{10} + \Gamma^{2}_{01}\Gamma^{1}_{20} + \Gamma^{3}_{01}\Gamma^{1}_{30} - \Gamma^{0}_{00}\Gamma^{1}_{01} - \Gamma^{1}_{00}\Gamma^{1}_{11} - \Gamma^{2}_{00}\Gamma^{1}_{21} - \Gamma^{3}_{00}\Gamma^{1}_{31} \tag{3}$$

$$R^{2}_{002} = \Gamma^{2}_{02,0} - \Gamma^{2}_{00,2} + \Gamma^{0}_{02}\Gamma^{2}_{00} + \Gamma^{1}_{02}\Gamma^{2}_{10} + \Gamma^{2}_{02}\Gamma^{2}_{20} + \Gamma^{3}_{02}\Gamma^{2}_{30} - \Gamma^{0}_{00}\Gamma^{2}_{02} - \Gamma^{1}_{00}\Gamma^{2}_{12} - \Gamma^{2}_{00}\Gamma^{2}_{22} - \Gamma^{3}_{00}\Gamma^{2}_{32} \tag{4}$$

$$R^{3}_{003} = \Gamma^{3}_{03,0} - \Gamma^{3}_{00,3} + \Gamma^{0}_{03}\Gamma^{3}_{00} + \Gamma^{1}_{03}\Gamma^{3}_{10} + \Gamma^{2}_{03}\Gamma^{3}_{20} + \Gamma^{3}_{03}\Gamma^{3}_{30} - \Gamma^{0}_{00}\Gamma^{3}_{03} - \Gamma^{1}_{00}\Gamma^{3}_{13} - \Gamma^{2}_{00}\Gamma^{3}_{23} - \Gamma^{3}_{00}\Gamma^{3}_{33} \tag{5}$$

$$R^{0}_{110} = \Gamma^{0}_{10,1} - \Gamma^{0}_{11,0} + \Gamma^{0}_{10}\Gamma^{0}_{01} + \Gamma^{1}_{10}\Gamma^{0}_{11} + \Gamma^{2}_{10}\Gamma^{0}_{21} + \Gamma^{3}_{10}\Gamma^{0}_{31} - \Gamma^{0}_{11}\Gamma^{0}_{00} - \Gamma^{1}_{11}\Gamma^{0}_{10} - \Gamma^{2}_{11}\Gamma^{0}_{20} - \Gamma^{3}_{11}\Gamma^{0}_{30} \tag{6}$$

$$R^{1}_{111} = 0 \tag{7}$$

$$R^{2}_{112} = \Gamma^{2}_{12,1} - \Gamma^{2}_{11,2} + \Gamma^{0}_{12}\Gamma^{2}_{01} + \Gamma^{1}_{12}\Gamma^{2}_{11} + \Gamma^{2}_{12}\Gamma^{2}_{21} + \Gamma^{3}_{12}\Gamma^{2}_{31} - \Gamma^{0}_{11}\Gamma^{2}_{02} - \Gamma^{1}_{11}\Gamma^{2}_{12} - \Gamma^{2}_{11}\Gamma^{2}_{22} - \Gamma^{3}_{11}\Gamma^{2}_{32} \tag{8}$$

$$R^{3}_{113} = \Gamma^{3}_{13,1} - \Gamma^{3}_{11,3} + \Gamma^{0}_{13}\Gamma^{3}_{01} + \Gamma^{1}_{13}\Gamma^{3}_{11} + \Gamma^{2}_{13}\Gamma^{3}_{21} + \Gamma^{3}_{13}\Gamma^{3}_{31} - \Gamma^{0}_{11}\Gamma^{3}_{03} - \Gamma^{1}_{11}\Gamma^{3}_{13} - \Gamma^{2}_{11}\Gamma^{3}_{23} - \Gamma^{3}_{11}\Gamma^{3}_{33} \tag{9}$$

$$R^{0}_{220} = \Gamma^{0}_{20,2} - \Gamma^{0}_{22,0} + \Gamma^{0}_{20}\Gamma^{0}_{02} + \Gamma^{1}_{20}\Gamma^{0}_{12} + \Gamma^{2}_{20}\Gamma^{0}_{22} + \Gamma^{3}_{20}\Gamma^{0}_{32} - \Gamma^{0}_{22}\Gamma^{0}_{00} - \Gamma^{1}_{22}\Gamma^{0}_{10} - \Gamma^{2}_{22}\Gamma^{0}_{20} - \Gamma^{3}_{22}\Gamma^{0}_{30} \tag{10}$$

$$R^{1}_{221} = \Gamma^{1}_{21,2} - \Gamma^{1}_{22,1} + \Gamma^{0}_{21}\Gamma^{1}_{02} + \Gamma^{1}_{21}\Gamma^{1}_{12} + \Gamma^{2}_{21}\Gamma^{1}_{22} + \Gamma^{3}_{21}\Gamma^{1}_{32} - \Gamma^{0}_{22}\Gamma^{1}_{01} - \Gamma^{1}_{22}\Gamma^{1}_{11} - \Gamma^{2}_{22}\Gamma^{1}_{21} - \Gamma^{3}_{22}\Gamma^{1}_{31} \tag{11}$$

$$R^{2}_{222} = 0 \tag{12}$$

$$R^{3}_{223} = \Gamma^{3}_{23,2} - \Gamma^{3}_{22,3} + \Gamma^{0}_{23}\Gamma^{3}_{02} + \Gamma^{1}_{23}\Gamma^{3}_{21} + \Gamma^{2}_{23}\Gamma^{3}_{22} + \Gamma^{3}_{23}\Gamma^{3}_{32} - \Gamma^{0}_{22}\Gamma^{3}_{03} - \Gamma^{1}_{22}\Gamma^{3}_{13} - \Gamma^{2}_{22}\Gamma^{3}_{23} - \Gamma^{3}_{22}\Gamma^{3}_{33} \tag{13}$$

$$R^{0}_{330} = \Gamma^{0}_{30,3} - \Gamma^{0}_{33,0} + \Gamma^{0}_{30}\Gamma^{0}_{03} + \Gamma^{1}_{30}\Gamma^{0}_{13} + \Gamma^{2}_{30}\Gamma^{0}_{23} + \Gamma^{3}_{30}\Gamma^{0}_{33} - \Gamma^{0}_{33}\Gamma^{0}_{00} - \Gamma^{1}_{33}\Gamma^{0}_{10} - \Gamma^{2}_{33}\Gamma^{0}_{20} - \Gamma^{3}_{33}\Gamma^{0}_{30} \tag{14}$$

$$R^{1}_{331} = \Gamma^{1}_{31,3} - \Gamma^{1}_{33,1} + \Gamma^{0}_{31}\Gamma^{1}_{03} + \Gamma^{1}_{31}\Gamma^{1}_{13} + \Gamma^{2}_{31}\Gamma^{1}_{23} + \Gamma^{3}_{31}\Gamma^{1}_{33} - \Gamma^{0}_{33}\Gamma^{1}_{01} - \Gamma^{1}_{33}\Gamma^{1}_{11} - \Gamma^{2}_{33}\Gamma^{1}_{21} - \Gamma^{3}_{33}\Gamma^{1}_{31} \tag{15}$$

$$R^{2}_{332} = \Gamma^{2}_{32,3} - \Gamma^{2}_{33,2} + \Gamma^{0}_{32}\Gamma^{2}_{03} + \Gamma^{1}_{32}\Gamma^{2}_{13} + \Gamma^{2}_{32}\Gamma^{2}_{23} + \Gamma^{3}_{32}\Gamma^{2}_{33} - \Gamma^{0}_{33}\Gamma^{2}_{02} - \Gamma^{1}_{33}\Gamma^{2}_{12} - \Gamma^{2}_{33}\Gamma^{2}_{22} - \Gamma^{3}_{33}\Gamma^{2}_{32} \tag{16}$$

$$R^{3}_{333} = 0 \tag{17}$$

$$R^{0}_{010} = \Gamma^{0}_{00,1} - \Gamma^{0}_{01,0} + \Gamma^{0}_{00}\Gamma^{0}_{01} + \Gamma^{1}_{00}\Gamma^{0}_{11} + \Gamma^{2}_{00}\Gamma^{0}_{21} + \Gamma^{3}_{00}\Gamma^{0}_{31} - \Gamma^{0}_{01}\Gamma^{0}_{00} - \Gamma^{1}_{01}\Gamma^{0}_{10} - \Gamma^{2}_{01}\Gamma^{0}_{20} - \Gamma^{3}_{01}\Gamma^{0}_{30} \tag{18}$$

$$R^{1}_{011} \equiv 0 \tag{19}$$

$$R^{2}_{012} = \Gamma^{2}_{02,1} - \Gamma^{2}_{01,2} + \Gamma^{0}_{02}\Gamma^{2}_{01} + \Gamma^{1}_{02}\Gamma^{2}_{11} + \Gamma^{2}_{02}\Gamma^{2}_{21} + \Gamma^{3}_{02}\Gamma^{2}_{31} - \Gamma^{0}_{01}\Gamma^{2}_{02} - \Gamma^{1}_{01}\Gamma^{2}_{12} - \Gamma^{2}_{01}\Gamma^{2}_{22} - \Gamma^{3}_{01}\Gamma^{2}_{32} \tag{20}$$

$$R^{3}_{013} = \Gamma^{3}_{03,1} - \Gamma^{3}_{01,3} + \Gamma^{0}_{03}\Gamma^{3}_{01} + \Gamma^{1}_{03}\Gamma^{3}_{11} + \Gamma^{2}_{03}\Gamma^{3}_{21} + \Gamma^{3}_{03}\Gamma^{3}_{31} - \Gamma^{0}_{01}\Gamma^{3}_{03} - \Gamma^{1}_{01}\Gamma^{3}_{13} - \Gamma^{2}_{01}\Gamma^{3}_{23} - \Gamma^{3}_{01}\Gamma^{3}_{33} \tag{21}$$

$$R^0_{020} = \Gamma^0_{00,2} - \Gamma^0_{02,0} + \Gamma^0_{00}\Gamma^0_{02} + \Gamma^1_{00}\Gamma^0_{12} + \Gamma^2_{00}\Gamma^0_{22} + \Gamma^3_{00}\Gamma^0_{32} - \Gamma^0_{02}\Gamma^0_{00} - \Gamma^1_{02}\Gamma^0_{10} - \Gamma^2_{02}\Gamma^0_{20} - \Gamma^3_{02}\Gamma^0_{30} \quad (22)$$

$$R^1_{021} = \Gamma^1_{01,2} - \Gamma^1_{02,1} + \Gamma^0_{01}\Gamma^1_{02} + \Gamma^1_{01}\Gamma^1_{12} + \Gamma^2_{01}\Gamma^1_{22} + \Gamma^3_{01}\Gamma^1_{32} - \Gamma^0_{02}\Gamma^1_{01} - \Gamma^1_{02}\Gamma^1_{11} - \Gamma^2_{02}\Gamma^1_{21} - \Gamma^3_{02}\Gamma^1_{31} \quad (23)$$

$$R^2_{022} \equiv 0 \quad (24)$$

$$R^3_{023} = \Gamma^3_{03,2} - \Gamma^3_{02,3} + \Gamma^0_{03}\Gamma^3_{02} + \Gamma^1_{03}\Gamma^3_{12} + \Gamma^2_{03}\Gamma^3_{22} + \Gamma^3_{03}\Gamma^3_{32} - \Gamma^0_{02}\Gamma^3_{03} - \Gamma^1_{02}\Gamma^3_{13} - \Gamma^2_{02}\Gamma^3_{23} - \Gamma^3_{02}\Gamma^3_{33} \quad (25)$$

$$R^0_{030} = \Gamma^0_{00,3} - \Gamma^0_{03,0} + \Gamma^0_{00}\Gamma^0_{03} + \Gamma^1_{00}\Gamma^0_{13} + \Gamma^2_{00}\Gamma^0_{23} + \Gamma^3_{00}\Gamma^0_{33} - \Gamma^0_{03}\Gamma^0_{00} - \Gamma^1_{03}\Gamma^0_{10} - \Gamma^2_{03}\Gamma^0_{20} - \Gamma^3_{03}\Gamma^0_{30} \quad (26)$$

$$R^1_{031} = \Gamma^1_{01,3} - \Gamma^1_{03,1} + \Gamma^0_{01}\Gamma^1_{03} + \Gamma^1_{01}\Gamma^1_{13} + \Gamma^2_{01}\Gamma^1_{23} + \Gamma^3_{01}\Gamma^1_{33} - \Gamma^0_{03}\Gamma^1_{01} - \Gamma^1_{03}\Gamma^1_{11} - \Gamma^2_{03}\Gamma^1_{21} - \Gamma^3_{03}\Gamma^1_{31} \quad (27)$$

$$R^2_{032} = \Gamma^2_{02,3} - \Gamma^2_{03,2} + \Gamma^0_{02}\Gamma^2_{03} + \Gamma^1_{02}\Gamma^2_{13} + \Gamma^2_{02}\Gamma^2_{23} + \Gamma^3_{02}\Gamma^2_{33} - \Gamma^0_{03}\Gamma^2_{02} - \Gamma^1_{03}\Gamma^2_{12} - \Gamma^2_{03}\Gamma^2_{22} - \Gamma^3_{03}\Gamma^2_{32} \quad (28)$$

$$R^3_{033} \equiv 0 \quad (29)$$

$$R^0_{120} = \Gamma^0_{01,2} - \Gamma^0_{12,0} + \Gamma^0_{10}\Gamma^0_{02} + \Gamma^1_{1o}\Gamma^0_{12} + \Gamma^2_{10}\Gamma^0_{22} + \Gamma^3_{10}\Gamma^0_{32} - \Gamma^0_{12}\Gamma^o_{00} - \Gamma^1_{12}\Gamma^0_{10} - \Gamma^2_{12}\Gamma^0_{20} - \Gamma^3_{12}\Gamma^0_{30} \quad (30)$$

$$R^1_{121} = \Gamma^1_{11,2} - \Gamma^1_{12,1} + \Gamma^0_{11}\Gamma^1_{02} + \Gamma^1_{11}\Gamma^1_{12} + \Gamma^2_{11}\Gamma^1_{22} + \Gamma^3_{11}\Gamma^1_{32} - \Gamma^0_{12}\Gamma^1_{01} - \Gamma^1_{12}\Gamma^1_{11} - \Gamma^2_{12}\Gamma^1_{21} - \Gamma^3_{12}\Gamma^1_{31} \quad (31)$$

$$R^1_{121} \equiv 0 \quad (32)$$

$$R^3_{123} = \Gamma^3_{13,2} - \Gamma^3_{12,3} + \Gamma^0_{13}\Gamma^3_{02} + \Gamma^1_{13}\Gamma^3_{12} + \Gamma^2_{13}\Gamma^3_{22} + \Gamma^3_{13}\Gamma^3_{32} - \Gamma^0_{12}\Gamma^3_{03} - \Gamma^1_{12}\Gamma^3_{13} - \Gamma^2_{12}\Gamma^3_{23} - \Gamma^3_{12}\Gamma^3_{33} \quad (33)$$

$$R^0_{130} = \Gamma^0_{10,3} - \Gamma^0_{13,0} + \Gamma^0_{10}\Gamma^0_{03} + \Gamma^1_{10}\Gamma^0_{13} + \Gamma^2_{10}\Gamma^0_{23} + \Gamma^3_{10}\Gamma^0_{33} - \Gamma^0_{13}\Gamma^0_{00} - \Gamma^1_{13}\Gamma^0_{10} - \Gamma^2_{13}\Gamma^0_{20} - \Gamma^3_{13}\Gamma^0_{30} \quad (34)$$

$$R^1_{131} = \Gamma^1_{11,3} - \Gamma^1_{13,1} + \Gamma^0_{11}\Gamma^1_{03} + \Gamma^1_{11}\Gamma^1_{13} + \Gamma^2_{11}\Gamma^1_{23} + \Gamma^3_{11}\Gamma^1_{33} - \Gamma^0_{13}\Gamma^1_{01} - \Gamma^1_{13}\Gamma^1_{11} - \Gamma^2_{13}\Gamma^1_{21} - \Gamma^3_{13}\Gamma^1_{31} \quad (35)$$

$$R^2_{132} = \Gamma^2_{12,3} - \Gamma^2_{13,2} + \Gamma^0_{12}\Gamma^2_{03} + \Gamma^1_{12}\Gamma^2_{13} + \Gamma^2_{12}\Gamma^2_{23} + \Gamma^3_{12}\Gamma^2_{33} - \Gamma^0_{13}\Gamma^2_{02} - \Gamma^1_{13}\Gamma^2_{12} - \Gamma^2_{13}\Gamma^2_{22} - \Gamma^3_{13}\Gamma^2_{32} \quad (36)$$

$$R^2_{132} \equiv 0 \quad (37)$$

$$R^0_{230} = \Gamma^0_{20,3} - \Gamma^0_{23,0} + \Gamma^0_{20}\Gamma^0_{03} + \Gamma^1_{20}\Gamma^0_{13} + \Gamma^2_{20}\Gamma^0_{23} + \Gamma^3_{20}\Gamma^0_{33} - \Gamma^0_{23}\Gamma^0_{00} - \Gamma^1_{23}\Gamma^0_{10} - \Gamma^2_{23}\Gamma^0_{20} - \Gamma^3_{23}\Gamma^0_{30} \quad (38)$$

$$R^1_{231} = \Gamma^1_{21,3} - \Gamma^1_{23,1} + \Gamma^0_{21}\Gamma^1_{03} + \Gamma^1_{21}\Gamma^1_{13} + \Gamma^2_{21}\Gamma^1_{23} + \Gamma^3_{21}\Gamma^1_{33} - \Gamma^0_{23}\Gamma^1_{01} - \Gamma^1_{23}\Gamma^1_{11} - \Gamma^2_{23}\Gamma^1_{21} - \Gamma^3_{23}\Gamma^1_{31} \quad (39)$$

$$R^2_{232} = \Gamma^2_{22,3} - \Gamma^2_{23,2} + \Gamma^0_{22}\Gamma^2_{03} + \Gamma^1_{22}\Gamma^2_{13} + \Gamma^2_{22}\Gamma^2_{23} + \Gamma^3_{22}\Gamma^2_{33} - \Gamma^0_{23}\Gamma^2_{02} - \Gamma^1_{23}\Gamma^2_{12} - \Gamma^2_{23}\Gamma^2_{22} - \Gamma^3_{23}\Gamma^2_{32} \quad (40)$$

$$R^3_{233} \equiv 0 \quad (41)$$

These are formulae for the independent components of the Riemann curvature tensor in all orthogonal curvilinear coordinates, and for all metric tensors. In this paper, we apply these formulae to derive the Riemann curvature tensor in the cartesian coordinate using the golden metric tensor. We also demonstrate how to use the Riemann curvature tensor in the cartesian coordinate to generate the hitherto unknown but mathematically most elegant, physically most natural Riemannian curvature scalar and ricci curvature tensor in the cartesian coordinate. it is imperative to say that a comma indicate a partial differentiation with respect to the unit vector. Thus (1, 2, 3, 0) denotes partial differentiation with respect to (x, y, z, ct).

2. THEORY

The relationship between the spherical polar coordinates (r, θ , φ x^0) and the Cartesian coordinates (x, y, z, x^0) are given by [2].

$$x = rsin\theta cos\varphi \quad (42)$$

$$y = rsin\theta sin\varphi \quad (43)$$

$$z = rcos\varphi \quad (44)$$

Where (x, y, z, x^0) have their usual meanings.

The Golden Metric tensor for all gravitational fields in nature is given in the spherical polar coordinates (r, θ, φ x^0) as [1].

$$x^0\text{=ct} \quad (45)$$

$$g_{11} = \left(1 + \frac{2}{c^2}f\right)^{-1} \quad (46)$$

$$g_{22} = r^2\left(1 + \frac{2}{c^2}f\right)^{-1} \quad (47)$$

$$g_{33} = r^2 sin^2\theta\left(1 + \frac{2}{c^2}f\right)^{-1} \quad (48)$$

$$g_{00} = -\left(1 + \frac{2}{c^2}f\right) \quad (49)$$

$$g_{\mu v} = 0;\ \ otherwise \quad (50)$$

This metric tensor will have to be transformed into the cartesian coordinate using the transformation relation [3];

$$\overline{g_{qs}} = \frac{\partial x^q}{\partial \bar{x}^q}\frac{\partial x^s}{\partial \bar{x}^s} g_{qs} \quad (51)$$

Hence the golden metric tensor in the cartesian coordinates are expressed as:

$$g_{11} = \left(1 + \frac{2}{c^2}f\right)^{-1} \quad (52)$$

$$g_{22} = \left(1 + \frac{2}{c^2}f\right)^{-1} \quad (53)$$

$$g_{33} = \left(1 + \frac{2}{c^2}f\right)^{-1} \quad (54)$$

$$g_{00} = -\left(1 + \frac{2}{c^2}f\right) \quad (55)$$

$$g_{\mu v} = 0;\ \ otherwise \quad (56)$$

Therefore, the contravariant tensor of the metric tensor is given as;

$$g^{11} = \left(1 + \frac{2}{c^2}f\right) \quad (57)$$

$$g^{22} = \left(1 + \frac{2}{c^2}f\right) \quad (58)$$

$$g^{33} = \left(1 + \frac{2}{c^2}f\right) \quad (59)$$

$$g^{00} = -\left(1 + \frac{2}{c^2}f\right)^{-1} \quad (60)$$

$$g_{\mu v} = 0; otherwise$$

2.1 Golden Riemann Curvature Tensor in Cartesian Coordinate

Based upon the golden metric tensor for all gravitational fields in nature, the components of the christoffel symbols of the second kind pseudo tensor are well known [4]. Therefore the independent components of the Riemann curvature tensor (1) to (41) are given in terms of the gravitational scalar potential and the Cartesian coordinate of space time as follows:

$$R^0_{000} \equiv 0 \quad (61)$$

$$R^1_{001} = -\left[\frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,0}\right]_{,0} - \left[\frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right) f_{,1}\right]_{,1} + \frac{2}{c^4}f_{,1}f_{,1} + \frac{1}{c^4}f_{,2}f_{,2} + \frac{1}{c^4}f_{,3}f_{,3} \quad (62)$$

$$R^2_{002} = -\left[\frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,0}\right]_{,0} - \frac{1}{c^2}\left[\left(1 + \frac{2}{c^2}f\right) f_{,2}\right]_{,2} + \frac{2}{c^4}\left(1 + \frac{2}{c^2}f\right)^{-2} f_{,0}f_{,0} + \frac{1}{c^4}f_{,1}f_{,1} + \frac{1}{c^4}f_{,3}f_{,3} + \frac{2}{c^2}f_{,2}f_{,2} \quad (63)$$

$$R^3_{003} = -\left[\frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,0}\right]_{,0} - \left[\frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right) f_{,3}\right]_{,3} + \frac{2}{c^4}f_{,3}f_{,3} + \frac{2}{c^4}\left(1 + \frac{2}{c^2}f\right)^{-2} f_{,0}f_{,0} + \frac{1}{c^4}f_{,1}f_{,1} + \frac{1}{c^4}f_{,2}f_{,2} \quad (64)$$

$$R^0_{110} = \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}\right]_{,1} - \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-3} f_{,0}\right]_{,0} + \frac{2}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,1}f_{,1} + \frac{2}{c^4}\left(1+\frac{2}{c^2}f\right)^{-4} f_{,0}f_{,0} - \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,2}f_{,2} - \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,3}f_{,3} \quad (65)$$

$$R^2_{112} = -\left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}\right]_{,1} - \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2}\right]_{,2} - \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-4} f_{,0}f_{,0} + \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,3}f_{,3} \quad (66)$$

$$R^3_{113} = -\left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}\right]_{,1} - \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3}\right]_{,3} - \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-4} f_{,0}f_{,0} + \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,2}f_{,2} \quad (67)$$

$$R^0_{220} = \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2}\right]_{,2} + \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-3} f_{,0}\right]_{,0} + \frac{2}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,2}f_{,2} + \frac{2}{c^4}\left(1+\frac{2}{c^2}f\right)^{-4} f_{,0}f_{,0} - \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,1}f_{,1} - \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,3}f_{,3} \quad (68)$$

$$R^1_{221} = -\left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2}\right]_{,2} - \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}\right]_{,1} - \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-4} f_{,0}f_{,0} + \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,3}f_{,3} \quad (69)$$

$$R^3_{223} = -\left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2}\right]_{,2} - \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3}\right]_{,3} - \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-4} f_{,0}f_{,0} + \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,1}f_{,1} \quad (70)$$

$$R^0_{330} = \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3}\right]_{,3} + \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-3} f_{,0}\right]_{,0} + \frac{2}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,3}f_{,3} + \frac{2}{c^4}\left(1+\frac{2}{c^2}f\right)^{-4} f_{,0}f_{,0} - \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,1}f_{,1} - \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,2}f_{,2} \quad (71)$$

$$R^1_{331} = -\left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3}\right]_{,3} - \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}\right]_{,1} - \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-4} f_{,0}f_{,0} - \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,2}f_{,2} \quad (72)$$

$$R^2_{332} = -\left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3}\right]_{,3} - \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2}\right]_{,2} - \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-4} f_{,0}f_{,0} + \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,1}f_{,1} \quad (73)$$

$$R^0_{010} \equiv 0 \quad (74)$$

$$R^2_{012} = -\left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,0}\right]_{,1} + \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,0}f_{,1} \quad (75)$$

$$R^3_{013} = -\left[-\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,0}\right]_{,1} + \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,1}f_{,0} \quad (76)$$

$$R^0_{020} = \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,0}\right]_{,2} - \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3}\right]_{,0} \quad (77)$$

$$R^1_{021} = -\left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,0}\right]_{,2} + \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,0}f_{,0} \quad (78)$$

$$R^3_{023} = -\left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,0}\right]_{,0} + \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,2}f_{,0} \quad (79)$$

$$R^1_{031} = -\left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,0}\right]_{,3} + \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,0}f_{,3} \quad (80)$$

$$R^2_{032} = -\left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,0}\right]_{,3} + \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,0}f_{,3} \quad (81)$$

$$R^0_{120} = \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}\right]_{,2} + \frac{3}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,1}f_{,2} \quad (82)$$

$$R^3_{123} = -\left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}\right]_{,2} + \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,1}f_{,2} \quad (83)$$

$$R^0_{130} = \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}\right]_{,3} + \frac{3}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,1}f_{,3} \quad (84)$$

$$R^2_{132} = -\left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}\right]_{,3} - \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,1}3 \quad (85)$$

$$R^0_{230} = \left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2}\right]_{,3} + \frac{3}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,3}f_{,2} \quad (86)$$

$$R^1_{231} = -\left[\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2}\right]_{,3} - \frac{1}{c^4}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,2}f_{,3} \quad (87)$$

2.2 Golden Ricci Curvature Tensor

Our aim here is to recall the definition of the Ricci curvature tensor from the theory of tensor analysis and demonstrate how to express it in terms of the golden metric tensor for all gravitational fields in nature in the Cartesian coordinate.

The Ricci curvature tensor in 4-dimensional space-time, $R_{\mu v}$ is defined in all gravitational fields and all orthogonal curvilinear coordinates x^α by

$$R_{\mu v} = R^{\epsilon}_{\mu v \epsilon} \quad (88)$$

And explicitly as

$$R_{\mu v} = R^1_{\mu v 1} + R^2_{\mu v 2} + R^3_{\mu v 3} + R^0_{\mu v 0} \quad (89)$$

$R^{\epsilon}_{\mu v \epsilon}$ is the Riemann's curvature tensor. This can be used for the expression of the Riemannian linear acceleration vector in all 4-dimensions in all orthogonal curvilinear coordinates and gravitational fields, based upon the golden metric tensor for all gravitational fields.

2.3 Ricci Curvature Tensor in Cartesian Coordinate

Given the general definition of the Ricci curvature tensor for all gravitational fields and all orthogonal curvilinear coordinates. We can now conveniently express the components of the Ricci curvature tensor based upon the golden metric tensor in Cartesian coordinate as follows:

$$R_{00} = R^0_{000} + R^1_{001} + R^2_{002} + R^3_{003} \quad (90)$$

$$R_{11} = R^0_{110} + R^1_{111} + R^2_{112} + R^3_{113} \quad (91)$$

$$R_{22} = R^0_{220} + R^1_{221} + R^2_{222} + R^3_{223} \quad (92)$$

$$R_{33} = R^0_{330} + R^1_{331} + R^2_{332} + R^3_{333} \quad (93)$$

It should be noted that

$$R^0_{000} = R^1_{111} = R^2_{222} = R^3_{333} = 0 \quad (94)$$

Explicitly,

$$R_{00} = -3[\frac{1}{C^2}(1+\frac{2}{C^2}f)^{-1}f_{,0}]_{,0} - [\frac{1}{C^2}\left(1+\frac{2}{C^2}f\right)f_{,1}]_{,1} - \frac{1}{C^2}[\left(1+\frac{2}{C^2}f\right)f_{,2}]_{,2} - [\frac{1}{C^2}\left(1+\frac{2}{C^2}f\right)f_{,3}]_{,3} + \frac{4}{c^4}f_{,1}f_{,1} + \frac{4}{c^4}f_{,2}f_{,2} + \frac{4}{c^4}f_{,3}f_{,3} + \frac{4}{c^4}(1+\frac{2}{c^2}f)^{-2}f_{,0}f_{,0} \quad (95)$$

$$R_{11} = -[\frac{1}{c^2}(1+\frac{2}{c^2}f)^{-1}f_{,1}]_{,1} - [\frac{1}{c^2}(1+\frac{2}{c^2}f)^{-3}f_{,0}]_{,0} - [\frac{1}{c^2}(1+\frac{2}{c^2}f)^{-1}f_{,2}]_{,2} - [\frac{1}{c^2}(1+\frac{2}{c^2}f)^{-1}f_{,3}]_{,3} + \frac{2}{c^4}(1+\frac{2}{c^2}f)^{-2}f_{,1}f_{,1} \quad (96)$$

$$R_{22} = [\frac{1}{c^2}(1+\frac{2}{c^2}f)^{-3}f_{,0}]_{,0} - [\frac{1}{c^2}(1+\frac{2}{c^2}f)^{-1}f_{,1}]_{,1} - [\frac{1}{c^2}(1+\frac{2}{c^2}f)^{-1}f_{,2}]_{,2} - [\frac{1}{c^2}(1+\frac{2}{c^2}f)^{-1}f_{,3}]_{,3} + \frac{2}{c^4}(1+\frac{2}{c^2}f)^{-2}f_{,2}f_{,2} \quad (97)$$

$$R_{33} == [\frac{1}{c^2}(1+\frac{2}{c^2}f)^{-3}f_{,0}]_{,0} - [\frac{1}{c^2}(1+\frac{2}{c^2}f)^{-1}f_{,1}]_{,1} - [\frac{1}{c^2}(1+\frac{2}{c^2}f)^{-1}f_{,2}]_{,2} - [\frac{1}{c^2}(1+\frac{2}{c^2}f)^{-1}f_{,3}]_{,3} - \frac{2}{c^4}(1+\frac{2}{c^2}f)^{-2}f_{,2}]_{,2} - \frac{2}{c^4}(1+\frac{2}{c^2}f)^{-2}f_{,3}f_{,3} \quad (98)$$

2.4 Golden Riemann Curvature Scalar

We recall the definition of Riemann curvature scalar from the theory of tensor analysis and show how to express it in terms of the golden metric tensor for all gravitational fields in nature in the cartesian coordinate.

By the theory of tensor analysis, the Riemann curvature scalar in 4-dimensional space, R, is given in all gravitational fields in all orthogonal curvilinear coordinates x^{μ} by

$$R = g^{\alpha\beta}R_{\alpha\beta} \quad (99)$$

Where $g^{\alpha\beta}$ is the metric tensor and $R_{\alpha\beta}$ is the ricci curvature tensor. This invariant quantity is known as the Riemann curvature scalar. This definition can therefore be used to express Riemannian curvature scalar in all 4-dimensions in all orthogonal curvilinear coordinate and gravitational fields based upon the golden metric tensor for all gravitational fields.

It therefore follows that the unique expression for the Riemann curvature scalar for all gravitational fields in nature in the cartesian coordinate is given as

$$R = g^{00}R_{00} + g^{11}R_{11} + g^{22}R_{22} + g^{33}R_{33} \quad (100)$$

Hence, simplifying (95), (96), (97), (98) and substituting in (100)

$$R = \frac{4}{c^2}\left(1+\frac{2}{c^2}f\right)^{-2} f_{,00} - \frac{16}{c^4}\left(1+\frac{2}{c^2}f\right)^{-3} f_{,0}f_{,0} - \frac{2}{c^2}f_{,11} - \frac{2}{c^2}f_{,22} - \frac{2}{c^2}f_{,33} + \frac{6}{c^4}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}f_{,1} + \frac{4}{c^4}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2}f_{,2} + \frac{2}{c^4}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3}f_{,3} \quad (101)$$

3. RESULTS AND DISCUSSION

In this paper we showed how to formulate the formulae for the independent components of the Riemann curvature tensor in all orthogonal curvilinear co-ordinates, and for all metric tensors [(10) to (41)]. We applied the golden metric tensor to formulate the Riemann curvature tensor in the cartesian coordinate [(61) to (87)]. Equations (95) to (98) are the golden ricci curvature tensor in the cartesian coordinates and equations (101) is called the golden Riemann curvature scalar in the cartesian coordinate. These results so obtained in this paper are hitherto unknown but mathematically most sound and elegant and physically most natural and radial satisfactory correction of all orders of c^{-2}. This is another exploitation of the application of the golden metric tensor to the Riemannian geometry and its application in theoretical physics.

4. CONCLUSION

The door is henceforth opened for the expression of Riemann curvature tensor, Riemann ricci curvature tensor and Riemann curvature scalar in all dimensions in all orthogonal curvilinear coordinates and all gravitational fields, based upon the golden metric tensor for all gravitational fields in nature.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Howusu SXK. Riemannian revolutions in mathematics and physics II. Anyigba: Kogi State University; 2009.
2. Omonile JF, Koffa DJ, Howusu SXK. Riemannian Laplacian in cartesian coordinate using great metric tensor of all gravitational fields in nature in cartesian coordinate as the fundamental quantities of Riemannian geometry. African Journal of Science and Research. 2015;4(3):26-28.
3. Howusu SXK. Vector and tensor analysis. Jos: Jos University Press Limited; 2003.
4. Chifu EN, Howusu SXK, Seydou H, Lumbi LW. Motion of particles of non-zero rest masses exterior to astrophysically real or hypothetical spherical distributions of mass whose tensor field varies with polar angle only. Science World Journal. 2008;3(2):117-119.

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